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Deterministic chaos and forecasting in Amazon's share prices

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Abstract

Research background: The application of non-linear analysis and chaos theory modelling on financial time series in the discipline of Econophysics.
Purpose of the article: The main aim of the article is to identify the deterministic chaotic behavior of stock prices with reference to Amazon using daily data from Nasdaq-100.
Methods: The paper uses nonlinear methods, in particular chaos theory modelling, in a case study exploring and forecasting the daily Amazon stock price.
Findings & Value added: The results suggest that the Amazon stock price time series is a deterministic chaotic series with a lot of noise. We calculated the invariant parameters such as the maxi-mum Lyapunov exponent as well as the correlation dimension, managed a two-days-ahead forecast through phase space reconstruction and a grouped data handling method.

Introduction

The "Black Monday" of Stock Markets in 1987 along with the one of 2008, are clear failures of the standard economic view with regard to the existing financial analytic tools. In 1995, Mantegna and Stanley (1995) introduced a new research field using the term "EconoPhysics". Adding to this field, a possible framework for modeling economic phenomena is presented based on nonlinear dynamics and chaos theory modeling.

The term chaos or chaotic system refers to a dynamic system that is sensitive to even small changes in its initial condition. This change is a natural process, but it is hard to be predicted or at least cannot be estimated using Newton's laws of physics. The first person to grasp the concept of chaos was the American mathematician and later meteorologist Edward Norton Lorenz in the 1960s. Lorenz's system is a set of common differential equations which for given values of the initial parameters, the system exhibits chaotic behavior. The attractor generated by these differential equations has the shape of a butterfly. The combination of the butterfly shape of the Lorentz attractor and the behavior of chaotic systems led to the term "butterfly effect". The term is used to describe small changes in the initial conditions (the flapping of a butterfly's wings in Asia) which may have an unpredictable result (creating a hurricane in the USA). Thus, chaos theory is a convenient approach for the examination of the attributes of financial data, since the behavior of the financial markets is influenced by several factors that are relative to the market and can be affected by both internal and external causes.

In cases when a financial system is a chaotic deterministic one, the knowledge of the degrees of freedom is crucial for its modelling. This information will help to achieve an out of sample prediction. The experimental data, especially those related to economic time series, are non-stationary and noisy. The measure of complexity is provided by correlating dimension and minimum embedding dimension, which makes the following processes difficult. The first provides information regarding the system's complexity while the latter provides degrees of freedom, i.e. the number of the independent variables that defines the system.

As a case study, the Amazon's stock price was chosen. This framework could have been applied to other economic time series as well. Amazon is a large company of new technology (new economy) and the behaviour of its stock price is of crucial importance to the index of the new Economy (Nasdaq-100). The goal of the present work is to study, for the first time, the application of a chaotic model on Amazon's stock daily prices, in order to make predictions as well as to detect extreme fluctuations in a short or long forecasting horizon.

The article composition is the following: next section gives the literature review, which confirms the applicability of "EconoPhysics" approach in finance and time series analysis. The next section describes the application of the non-linear analysis. The following part is devoted to forecasting the out of sample values, using the phase-space-reconstruction method and the Group Method of Data Handling (GMDH). Finally, in the last parts of the article the discussion and conclusions are given.

Literature review

During recent years, the prediction of the stock market prices, besides the interest from researchers specializing in linear forecasting methodologies, is the subject of interest of researchers with expertise in physics especially those specializing in non-linear modelling and chaos theory (Stanley *et al*., 1995; Kodba *et al*., 2005; Garas & Argyrakis, 2007; Kennett *et al*., 2010). For that purpose, a plethora of econometric models have been developed, offering linear forecasting methods mostly based on autoregressive processes. However, the returns of financial data are not characterized by the normal distribution, and linear models are not capable of capturing the complex structure of financial data, thus resulting in poor performance. More recent research efforts have aimed at examining the non-linear structure of financial time series, using a multitude of different approaches based on chaos theory in order to achieve higher level of prediction accuracy.

Many physicists (Kennett *et al*., 2010; Garas & Argyrakis, 2007; Kodba *et al*., 2005) have published works using different theories from physics to study financial markets and even the macroeconomic behavior of countries (Magafas, 2013). The Grassberger and Procaccia method in the work of Schwartz and Yousefi (2013) was selected to calculate the correlation dimension and the minimum embedding dimension. Based on previous analysis, the next step is to apply these results to predict out of sample values of the corresponding time series with enough accuracy.

Diaz (2013) after conducting rescaled range analysis on four Dow Jones indices concluded that time series are persistent, and the Hurst exponent values exhibit long memory and therefore investors should cautious when using linear models for forecasting purposes.

Su *et al.* (2014), by performing non-linear analysis on agricultural time series, inferred that the data exhibit deterministic chaotic behaviour. The rescaled range analysis resulted to a Hurst exponent of value above 0.5, indicating the fractal structure of the data. The Largest Lyapunov exponents are positive, and the correlation dimension corresponds to a non-integer value. All these remarks highlight the existence of low dimensional chaos within the data and the possibility of short-term prediction.

Faggini (2014) suggested that if an economic index, for a certain time period, is modelled as a Brownian process or as a deterministic chaotic one, it can be contaminated by noise or not. Stavrinides *et al*. (2015) has proposed a circuit (physical system), implementing a financial system with time-delayed feedbacks. The simple form of this dynamic system, without any time-delayed feed, has been already investigated and was found to demonstrate both a periodic and a chaotic behavior. The control of the circuit-system's chaotic behavior could be achieved by introducing the time-delayed feedback. The overall operation was simulated, using NI's multiset and control of its behavior was achieved by controlling feedback delay-time of a certain variable corresponding to a financial variable.

Fan *et al.* (2017) studied a resource-economy-pollution dynamic system and chaotic relationship was obtained. This relationship indicts that a small change in economy growth may cause catastrophic environmental damage. Therefore, the importance of chaotic analysis was marked.

Lahmiri (2017) showed that business family stock returns are not chaotic, while market returns exhibit evidence of chaotic behaviour. The research also showed that most of family business stocks and market index exhibit long memory in volatility. Xu *et al.* (2018) studied a simple dynamic system of stock price time series' fluctuation based on the rule of stock market. When the stock price time series' fluctuation has been disturbed by external excitations, the system exhibited obviously chaotic phenomena, and its basic dynamic properties have been analysed.

In the research by Sahni (2018), the results obtained from machine learning, representing a substantial increase in returns over existing algorithmic trading engines is shown that this approach is modelled around chaos theory. In the study by Bildirici *et al.* (2019) Local Lyapunov exponent and Shannon Entropy tests were utilized in order to determine the chaotic behaviour of CDS for the USA, Turkey and China. According to the results, all CDS series have chaotic dynamics. The presented literature confirms the potential applications' of "EconoPhysics" perspective in exploring financial data.

The research: method and its application

Non-Linear Analysis

The daily Amazon stock value is presented as a time dependent variable x=x(t) as it is shown in Figure 1. It covers N=2758 data from 25/04/2006 to 07/04/2017, with the sampling rate Δt=1 day.

As mentioned before, the reconstruction of the corresponding possible strange attractor is based on Grassberger and Procaccia's method (Magafas *et al*., 2017; Hanias *et al*., 2008; 2009). Treating the Amazon stock price as a variable which belongs to a wider economic system means that it is subject to the influence of other unknown variables. It is crucial to know the number of differential equations of first order so the system can be modelled. Initially, this number is unknown because the number of other variables is unknown as well. Using Takens' theorem, (Takens, 1981), from one recorded time series a topological equivalent phase space can be constructed. In financial time series, it is difficult to separate the part of the time series that is a deterministic chaotic one and the part with random component and what kind of noise is produced by the randomness (Faggini, 2014).

According to Takens (1981) the correlation integral is calculated, (Hanias *et al*., 2008), for r→0 and N→ ∞ by equation (1) as follows:

$$C(r) = \frac{1}{N_{pairs}} \sum_{\substack{i=1,\\ j=i+1+W}}^{N} H\left(r - \left\|\vec{X}_i - \vec{X}_j\right\|\right) \tag{1}$$

where:
N – the number of points,
r – the linear dimension of hyperspheres,
H – the Heaviside function.

$$N_{pairs} = \frac{2}{(N-m+1)(N-m+W+1)},$$

where:
m – the embedding dimension,
$(\vec{X}_i, \vec{X}_j)$ – the number of pairs for which the distance, (Euclidean norm),
$\left\|\vec{X}_i - \vec{X}_j\right\|$ – less than r, in an m dimensional Euclidean space,
W – the Theiler window,
N – the number of data.

For the Amazon stock price time series N=2758 $\vec{X}_i$ is a vector which is determined as in equation (2):

$$\vec{X}_i = \{x_i, x_{i-\tau}, x_{i-2\tau}, \ldots x_{i+(m-1)\tau}\} \qquad (2)$$

where:
τ – the time delay τ = iΔt.

The time delay is estimated using the average mutual information function I(t) (Abarbanel, 1996; Fraser & Swinney, 1986). The average mutual information between x(t) and x(t+τ), is the amount in bits found by values of x(t) through measurements of x(t+τ) (Abarbanel, 1996). For example, when recording the daily temperature every second, the values are highly correlated contrary to 12-hour measurements which results in loss of information. In order to get optimal τ, the suggestion is to take the τ as the first minimum of the mutual information I, (Fraser & Swinney, 1986). For the Amazon time series, the first minimum is at τ=42-time steps as depicted in Figure 2. Of course, this value is a starting point. Practically the influence of τ values on correlation dimension estimation in this region of values is negligible.

The Theiler window, (Kantz, 1997) is estimated with the help of space time separation plots (stp). In Figure 3, the stp plots are shown. The temporal correlations are detected by plotting the spatial separations versus the orbit lag. From the saturation of curves (Kantz, 1997), the Theiler window is estimated to be W=160.

Using value τ=42 and W=160, the calculation of correlation integrals for various embedding dimensions is performed. According to Ott *et al.*, (1994), Abarbanel (1996), and Sprott (2003), if the relation between correlation integral and radius is in the form of a power law, then the attractor is a stranger one and v the correlation dimension.

The ln C(r) vs lnr for embedding dimensions m=1 to 10 is shown in Figure 4.

From the linear parts of the curves in Figure (4) the correlation dimension is estimated. In Figure 5 the average slopes v vs m is given for different embedding dimensions. The v saturates at value of v=2.28.

According to Abarbanel (1996), the minimum embedding dimension m_{min} is the next integer above the correlation dimension. This value is $m_{min}=3$. From Figure 5, the value of m where v reaches the plateau is 5, (Tassis *et al.*, 2017). The conclusion is that there are at least 3 essential variables that are enough to reconstruct the strange attractor, and the global dynamics are described by 5 variables. The distinction between low dimensional chaos and correlated (random) noise should not be based solely on correlation dimension estimates (Provenzale *et al.*, 1992). Other types of stochastic processes minimize the properties of low dimensional chaos in finite data sets (Weron, 2002). For this purpose, we analyze the first (numerically) derivative of time series. When this quantity increases as embedding dimension increases then no saturation is observed, and the signal is dominated by high dimensional white noise. Conversely, if the results of the analysis do not change under signal differentiation, the dynamic is a deterministic chaotic one and low dimensional. Nevertheless, if the derivative (Provenzale *et al.*, 1992) of the time series has an accurate plateau (correlation dimension — embedding dimension diagram) and the resulting correlation dimension is considerably larger than that of the original one, then the time series is considered a stochastic one. The first differences are displayed in Figure 6.

We denote the correlation dimension for the first difference time series as D_2. In Figure 7, we present the correlation dimension D_2 vs embedding dimension m.

The correlation dimension D_2 =3.62 is larger than the correlation dimension of the original time series which is found to be v=2.28. With correlation dimension D_2=3.62, the minimum embedding dimension is m_{min}=4, as said above, while the global dynamics needs 7 variables as Figure7 indicates. The difference between correlation dimensions of original data and first differential data indicates that the dynamics have a significant noisy component (Provenzale *et al.*, 1992). This component will prevent a reliable prediction.

Time series prediction

It is well known that many economic time series are non-stationary making the prediction procedure unreliable (Schwartz & Yousefi, 2013). In other words, every week the parameters as time delay correlation and embedding dimension change due to noise influence and to the lack of stationarity, so the analysis must be repeated. Additionally, for certain time and to investigate the trend of time series, the persistence of time series must be checked. The reliability of forecasting depends on the memory properties of the time series. If the time series is persistence large values tend to be followed by large values too. A perturbation will be influencing the future predictions for an exceptionally long time (Weron, 2002). The opposite effect happens in anti-persistence time series. The Hurst exponent measures the time series memory effect. To estimate the Hurst exponent a (R/S) analysis was applied. A Hurst exponent, H, between 0 and 0.5 corresponds to an anti-persistent time series, H=0.5 corresponds to random walk. H >= 0.5 corresponds to persistent time series. In case which H=0.5 the time series is essentially a random walk that cannot be predicted (Peters, 1991; 1994). The majority of time series possess Hurst exponent near 0.5, (Balakin *et al*., 2004), but there are examples of persistence time series as Dow Jones with H=0.72, (Peters, 1991) or anti-persistence as the volatility of the S&P composite prices with H=0.3 (Peters, 1991).

To estimate Hurst exponent, we apply the Rescaled Range Analysis, R/S analysis as mentioned before (Peters, 1991; 1994; Weron, 2002). The relationship is given in equation (3) giving the Hurst exponent H for a specific time series:

$$\frac{R}{S} = (an)^H \tag{3}$$

where:
R – the range of time series so $R=x_{max}-x_{min}$
x_{max} – the maximum value of x(t),
x_{min} – the minimum value of x(t),
S – the standard deviation of the original observation,
a – a constant,
n – the number of observations for sub time series,
H – the Hurst exponent.

By taking the log of equation (3), we obtain equation (4) (Peters, 1991; 1994; Weron, 2002):

$$\log\left(\frac{R}{S}\right) = Hlog(n) + \log(a) \tag{4}$$

Calculating the slope of log/log graph of R/S vs n will, therefore, give us an estimate of H. This is shown in Figure 8.

The Hurst exponent is estimated to be H=0.969. With this value the time series is a persistent one. Additionally, it is well known that fractal time series are characterized by long memory processes (Mantegna & Stanley, 1995).

The R/S analysis gives reliable results for stationary time series. If the series is not stationary, we apply the Detrended Fluctuation Analysis (DFA), (Peng *et al*., 1995). The advantage of DFA over R/S analysis is that overcomes the spurious detection of apparent long-range correlations that are an artifact of the non-stationarity. First the Amazon stock price time series of total length N is integrated according to equation (5):

$$y(k) = \sum_{t=1}^{k}[x(t) - x_{av}] \tag{5}$$

where:
x_{av} – the average value of x(t).

The time series are divided into bins of equal length n, the data are fitted with a least square for every bin. The corresponding line gives the trend in that bin. The detrending procedure is followed by subtracting the local trend $y_n(k)$ in each box from y(k). The root mean square fluctuation of this integrated and detrended time series is calculated by equation (6) (Peng *et al.*, 1995):

$$F(n) = \sqrt{\frac{1}{N}\sum_{k=1}^{N}[y(k) - y_n(k)]^2} \tag{6}$$

For all time scales which are represented with box sizes, this computation is repeated to provide a relationship between F(n) and n. Typically, the relationship between logF(n) and logn is linear indicating a power law. The slope α of the line characterizes the fluctuations.

The scaling exponents 'α' is classified as:

- Uncorrelated sequence: $\alpha \sim 0.5$
- Anti- correlated sequence: $0<\alpha<0.5$
- Long – range temporal correlations: $0.5<\alpha<1$
- Strong correlations that are not of a power law form: $\alpha>=1$, (Peng *et al.*, 1995).

The findings of the DFA analysis are depicted in Figure 9.

The exponent $\alpha=1.6$ is close to the value of random walk ($\alpha=1.5$) (Peng *et al.*, 1995). This means that the underlying dynamic is contaminated by noise. The DFA method reveals the trend and the additive fractal noise. This result agrees with the increase of correlation dimension of the first difference time series as mentioned in Section 3. This R/S and DFA analysis is typical for financial time series, but there is a need for a connection with the chaotic nature in order to investigate the predictability of that particular time series.

Keeping in mind that financial time series are contaminated by noise, the prediction horizon must be delimited. The Lyapunov exponent gives the predictability degree of time series. According to Sugihara and May (1990), with $\tau=42$ from Figure 10, we estimate the maximum Lyapunov to be $\lambda_{max}=$ 0.374. The positive Lyapunov exponent means that the system is unstable and chaotic, (Faggini, 2014). The value of $\lambda_{max}= 0.374$ means that the predictable horizon $1/\lambda_{max}$ is about 2 days ahead for the Amazon time series

(Weron, 2002). This value of maximum Lypunov exponent gives a strong restriction to predict further i.e. a horizon of 100 points ahead. The same restriction is applied to the first difference time series. In this case, the maximum Lypunov exponent is equal to 1.40 so the time horizon is strictly one day ahead.

Based on phase space reconstruction, following the trajectory of variable x as the attractor's stretches and folds a local simple prediction model, using the near neighbors, can give reasonable prediction results (Kantz, 1997; Magafas *et al*., 2017). From vector $\vec{X}_i$ we can construct an m-dimensional signal $\{x_i^m\} = \{(x_i, x_{i+\tau}, x_{i+2\tau}, ..., x_{i+(m-1)\tau})\} \in R^m$. From stretching and folding of the m-dimensional vector point $\{x_N^m\}$, into m phase space we can predict the time evolution of $\{x_{NN+k}^m\}$ k points if it is known how its neighborhood behaves using a local weighted least squares fitting over all neighbors' projections k-steps ahead (Sugihara & May, 1990; Peters, 1991; Schouten *et al.,* 1994; Hanias *et al.,* 2009; Thalassinos *et al*., 2009).

With τ=42 m = 3 and optimum number of neighbors, nn to be nn= 8 as a trial and error result, actual and forecast time series for k=1-time steps ahead are presented in Figure 11.

The out of sample forecast gives:

– Predicted value is 894.6 while the actual value is 894.88.

Trying to predict a next point was unsuccessful. However, using the global embedding dimension m=7 as the analysis of the first difference time series shown, we forecast successfully, out of sample, two days ahead. The parameters were m=7, nn=8 τ=42 and the results were:

– Predicted value 898.621 with actual value 898.28.
– Predicted value 893.907 with actual value 894.88.

Applying the method of Group method of Data Handling (GMDH), two steps ahead prediction is achieved (Ivakhenko, 1968; Ivakhenko & Ivakhenko, 1995). GMD algorithm is applied to predict stock prices as an effective approach in time series prediction (Fallahi *et al.*, 2001). The corresponding algorithms gradually generate complicating models and then select a set of models that show the highest forecasting accuracy at previously unseen data. Combinatorial GMDH model is a polynomial function with linear parameters. The final model consists of time series transformed into sets of lags. The lags are determined by a window size equal to the embedding dimension. These findings are illustrated in Figure 12. The last two points are the out of sample prediction points.

The out of sample forecast gives:

- Predicted value 898.621 with actual value 899.24.
- Predicted value 893.907 with actual value 895.75.

We calculate the Root Mean Square Error as a precision measurement of the fit of the model found to be 1.374738.

Discussion

In this article, we have classified the Amazon stock price time series as a deterministic chaotic one contaminated by noise for a specific period. By using the method of phase space reconstruction, a short time prediction of the Amazon prices was achieved, and the results have provided us with some additional information. It is observed that the predictor error increases along with the prediction interval. This is an indication of chaotic behavior caused by the separation between neighboring vectors within the phase space and is not met when the data follows a random walk process. The method of nonlinear analysis had been applied successfully as in the work of Ozun *et al.* (2010). The nonlinear analysis and the prediction of stock returns using Greek and Turkish stock index data had shown empirically whether the markets have informational efficiency, in a comparative perspective (Ozun *et al*., 2010). Similar results i.e deterministic chaotic behavior, have been found not only in individual stock analyses but also in the behavior of a more global indicator as the S&P index (Hanias *et al*., 2013). Additionally, the fact that economic systems obey deterministic laws and in a critical state had been proved (Ozun *et al*., 2014).

Additionally, the high level of persistence observed from the results of the DFA test highlights the fractal structure of the Amazon time series which exhibits long-term memory. Further investigation of the Hurst exponent could possibly identify non-periodic cycles within the data related to the business cycles of the company. The correlation dimension, estimated for both raw data and first difference time series forms of the data, shows in both cases a saturation level, which suggests a low dimensionality strange attractor. On the contrary, if the data were following a random walk process, the dimension of the phase space would continue to increase since Brownian noise attempts to fill all the space that is given to it.

This is a crucial test to distinguish between random data that mimic chaotic behavior and deterministic chaotic one. The positive values of the Local Lyapunov exponent indicates the exponential divergence between neighboring trajectories interpreting the sensitivity to initial conditions that verifies the existence of chaotic dynamics within the data. Of course, the positive Lyapunov exponent is an indicator only of determinism in data, because random walk can exhibit in some cases positive Lyapunov exponent (Provenzale *et al.*, 1992). The method of GMDH is a promising and robust tool in order to predict financial data (Zaychenko, 2008). In our case this is confirmed by predicting the Amazon share price time series with high accuracy.

Conclusions

The chaotic analysis of the Amazon' stock prices has shown that the time series is defined by low dimensional chaos, contaminated with noise. The time series is persistent as Hurst exponent suggests and exhibits strong correlations that are not a result of a power law as DFA analysis proves. Besides the limits of the non-stationarity, one and two steps ahead prediction is achieved using phase space reconstruction of the corresponding strange attractor.

Despite the encouraging results, we should not miss the fact that when the time series is non-stationary, as most of time series in economy (Peters, 1991), the change in mean and standard deviation complicates the task of forecasting both with the method of nearest neighbors via phase space reconstruction and with the method of GMDH. Due to non-stationary, as the system evolves in time, the parameters must be redefined for further forecast.

We can make safe forecasting for a short period of one day or two, but it is also important that we can detect large fluctuations. This can help investors to identify periods of high volatility and take it into consideration in their risk management plan. Future work could estimate the efficiency of long term forecasting by applying the methodology presented while using weekly prices as well further investigating the non-linear properties and dynamics of the dataset and the modification of the regression fitting in order to improve the precision of the prediction results. Additionally, in future work methods of noise reduction, especially nonlinear methods of noise reduction, would apply. This task would improve the quality of prediction increasing the prediction horizon.

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Annex

Figure 1. Time series of Amazon's stock values in USD

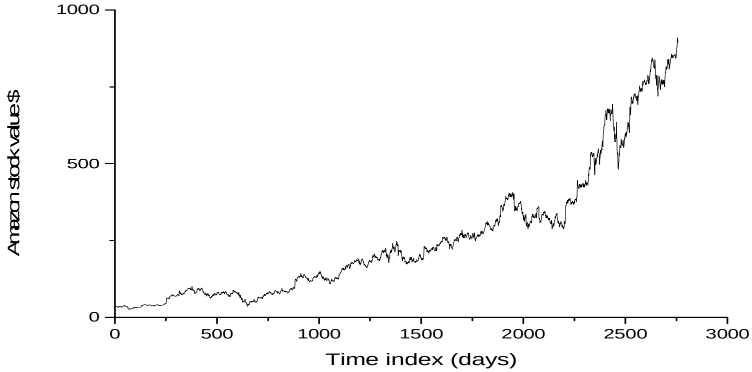


Figure 2. Mutual Information I vs time delay τ for Amazon' stock prices. The first minimum is at τ=42. The table shows the first minimum with detail.

τ	I(τ)
39	2.71308
40	2.70807
41	2.70562
42	2.69871
43	2.69952

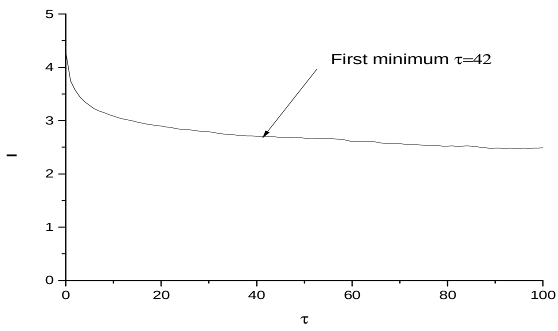


Figure 3. Space Time separation plots for Amazon Stock value. From the curve's plateau Theiler window estimated to be 160

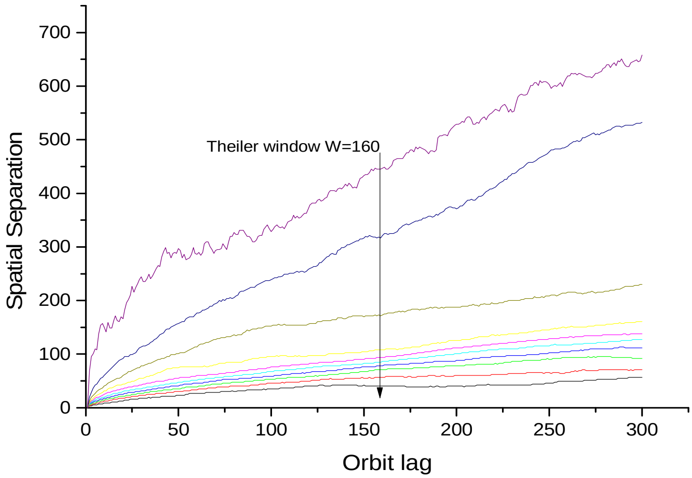


Figure 4. Correlation integral C(r) vs. radius r in ln–lnplot for various embedding dimensions m for the Amazon time series. The values of m are followed by a top to bottom sequence. From the slopes of linear parts of these curves, the correlation dimensions are estimated for various embedding dimensions m.

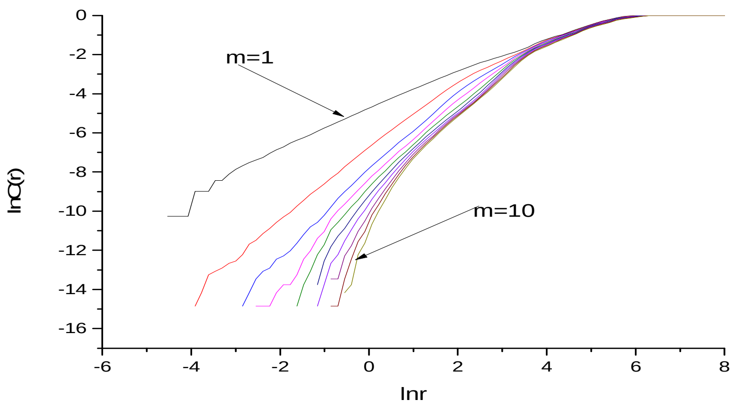


Figure 5. Correlation dimension v vs. embedding dimension m. From plateau, the saturation is achieved after the value of m=5. On the vertical axis, the correlation dimension v saturates at the non-integer value of v=2.28

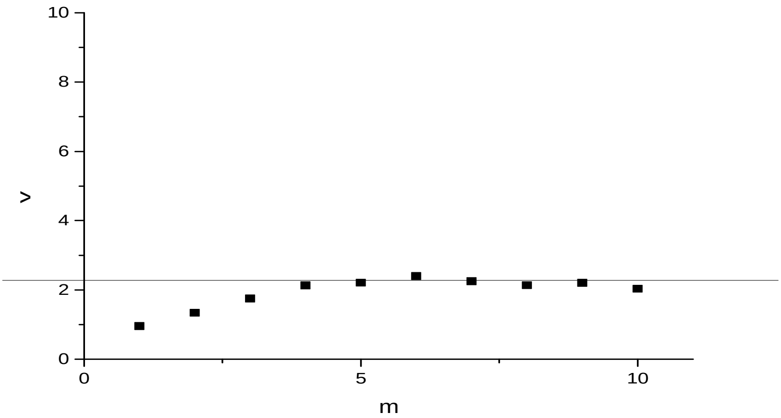


Figure 6. First differences of Amazon time series

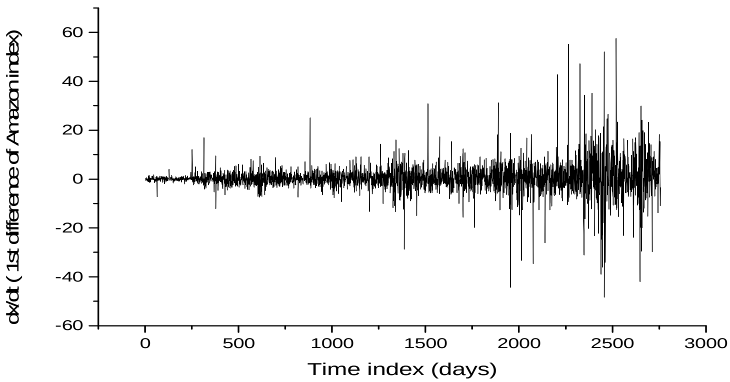


Figure 7. Correlation dimension D_2 vs. embedding dimension m for first difference time series of Amazon time series. The saturation level that forms a plateau, is achieved after the value of m=7. On the vertical axis, the correlation dimension v saturates at the non-integer value of v=3.62

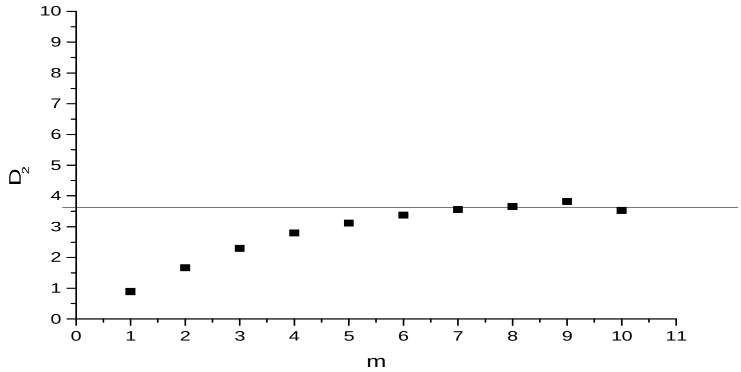


Figure 8. (R/S) Analysis for Amazon Stock value. From the slope Hurst exponent is estimated to be H=0.969

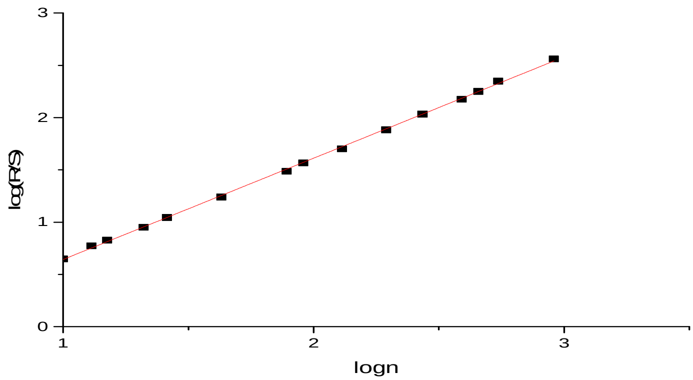


Figure 9. DFA Analysis for Amazon time series. From the slope exponent α is estimated to be α=1.6

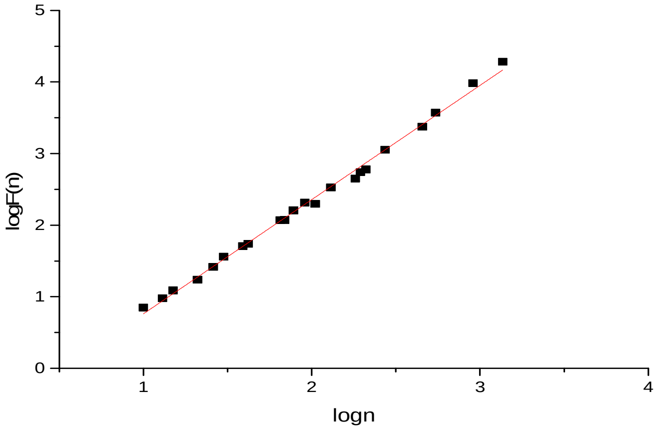


Figure 10. The Amazon's time series average local Lyapunov exponents for a phase space with embedding dimension m=3.

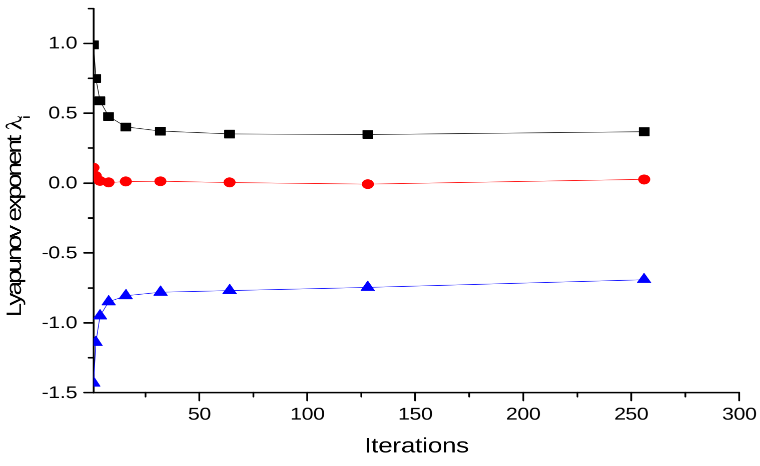


Figure11. Actual and predicted time series for k=1time steps ahead with m=3 nn=8 τ=42

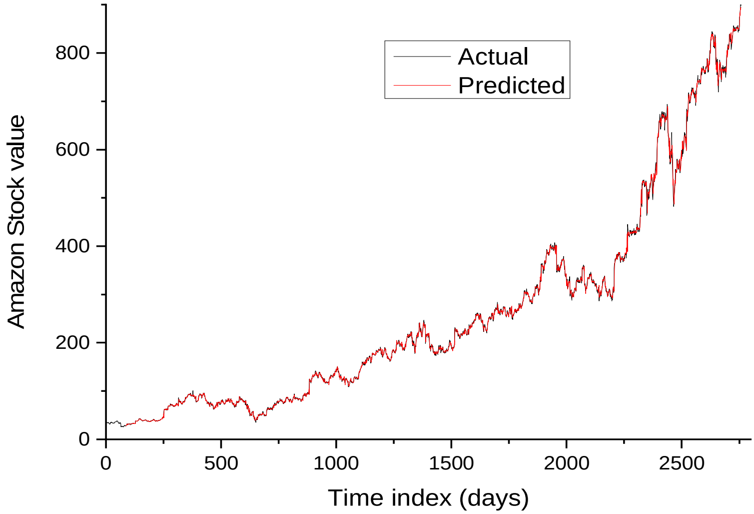
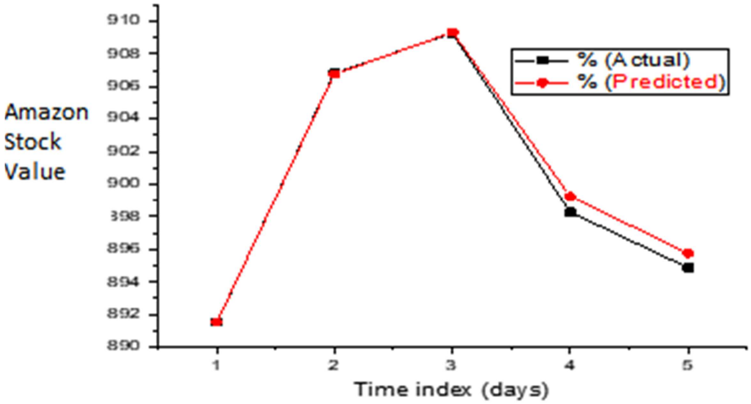


Figure 12. Actual and predicted time series with GMDH method



Note: The last two points are the out of sample prediction points.

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