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Economic analysis of implementing VMI model using game theory

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Abstract

Research background: The article deals with implementing VMI between the supplier and customer. To assess whether the system will be implemented, the evolution game theory is used. The contribution is based on the limitations of the study of the evolutionary game theory approach to modelling VMI policies (Torres *et al.*, 2014) and its later extension, The evolutionary game theory approach to modelling VMI policies (Torres & García-Díaz, 2018). It aims to complement the studies and provide a comprehensive picture of the issue.

Purpose of the article: The main objective of the contribution is to respond to the question whether the VMI system will be introduced between the supplier and customer.

Methods: In the first phase, the matrix is analysed from the point of view of the game meaning and its limit parameters. The limit parameters are set taking into account the economic reality. The only examined states of the matrix are those where the result is not obvious. For the purposes of the contribution, we work with a 5-year period. A new software capable of calculating evolutionary focus and their stability is created. Sensitivity analysis is carried out for the individual parameters that affect the system behaviour.

Findings & Value added: Value added is a complex description of the system and complementation of previous studies in this field. VMI is confirmed. The results obtained can be used for practical management, so that the managers are able to identify what the actual costs are and what the probability of introducing the system is. At the same time, they can identify the parameters that can be influenced by them and observe their impact on the shift of the system introduction probability.

Introduction

Vendor management inventory (hereinafter referred to as VMI) is a logistic system connecting the supplier and customer (Shen *et al.*, 2013). Due to this connection, there can be a possibility of reducing costs. Moreover, this connection can also have a positive impact on the environment (Bazan *et al.*, 2017). Connecting the supplier and customer is a demanding process; therefore, there is a possibility it will not occur (Prajogo & Olhager, 2012). The contribution investigates whether the connection will occur, which depends on a number of parameters, some of them being of great importance, while others only marginal. The objective of the contribution is to analyse such economic parameters and present them as an integrated system.

The contribution is based on the study entitled *An evolutionary game theory approach to modelling VMI policies* (Torres *et al.*, 2014) and its later extension, *An evolutionary game theory approach to modelling VMI policies* (Torres & García-Díaz, 2018). The first part of the practical chapter extends the analysis of the given issue, focusing mainly on the areas which the authors themselves describe as the limitations of the study.

Literature review

Supplier-customer integration

The integration of supplier-customer relations can enhance performance of individual companies. However, it is a demanding managerial process (Prajogo & Olhager, 2012). In order to introduce the system, it is necessary to integrate both material and information flows (Fisher, 1997). If the VMI system is introduced (Selldin & Olhager, 2007), it is possible to achieve

reduction of inventory and incidents related to goods shortage. In terms of a conventional supplier chain, the formula to calculate the overall goods storage and supply costs in the case of optimal supply (Torres *et al.*, 2014) for the customer is as follows:

$$TC_B = \sqrt{2rn_d n_s} \quad (1)$$

where:

n_s – costs of holding inventory (unit) expressed in q.

n_d – costs of one supply,

r – overall quantity per year,

For supplier:

$$TC_M = \sqrt{2rn_{dM}n_{sM} \left(1 - \frac{r}{p}\right)} \quad (2)$$

where:

n_{dM} – delivery costs for manufacturer,

n_{sM} – costs of holding for manufacturer,

q_m – supply volume

p – overall production.

For the VMI, optimized costs for customer are as follows:

$$TC_{B-VMI} = \frac{1}{2} \sqrt{\frac{2n'_d r}{n_s + n_{dM} \left(2\frac{r}{p} - 1\right)}} \left[2n_s + n_{dM} \left(2\frac{r}{p} - 1\right)\right] \quad (3)$$

where:

n'_d – costs of holding for the customer in the case of introduced VMI,

k – VMI coordination constant,

n_{dM} – costs of delivery for manufacturer,

n_{sM} – costs of holding for manufacturer,

q_m – supply volume,

p – overall production.

Optimized costs for the manufacturer are as follows:

$$TC_{M-VMI} = \sqrt{2n_{dM}n_{sM}r \left(1 - \frac{r}{p}\right)} + \frac{H}{2} \left(2\frac{r}{p} - 1\right) \sqrt{\frac{2n'_d r}{n_s + n_{dM} \left(2\frac{r}{p} - 1\right)}} \quad (4)$$

Evolutionary game theory

Evolutionary game theory is a suitable and modern tool for supply chain analysis (Chetna & Singh, 2018). Using evolutionary game theory enables to reach the balance as well as to determine its conditions and dynamics (Barari *et al.*, 2012). It is possible to explore not only the relations between companies, but also between other entities (Zhu & Dou, 2007). Of course, it is not the only tool possible; it is often used in combination with other tools (Tian *et al.*, 2014). Finally, game theory can be used also for a planning process (Bergantiños & Leticia, 2019), in computer applications (Debroy *et al.*, 2019).

The basis of evolutionary game theory was laid by Maynard and Price (1973), and later specified by Smith (1974). The authors focus on the concept of evolutionary stable strategy. This concept is explained on the example of animal conflicts over rare stuff (food, territory, etc.). The utility of the concept lies in the fact that it abandons the premise of perfect rationality. The theory thus gets closer to reality and it is easier to apply to human behaviour.

Evolutionary game theory is based on the principle of evolution (McKenzie, 2009), which, in its simplified form, is based on the fact that individuals of a given species adapt to the environment (they mutate). If the adaptation process is successful, the individual survives and passes the experience on to other generations, and this mutation spreads gradually on more and more individuals. Exploration of evolutionary game theory has a wide application and currently is a studied area (Carlos *et al.*, 2009). One of the most widely-known and analysed conflicts is a hawk-dove conflict (Smith, 1982).

Evolutionary game theory takes this theory by focusing on a population of individuals, who are game players. This population can be of different size and, depending on the size, it can influence the individual mutations of the population members. In this context, mutation refers to a change in the strategy compared to the majority of the population.

A great advantage of this theory is abandoning the premise of perfect informing of the individual players, which is a better representation of the real world. Each player watches whether the utility changes with the change of the strategy, and strives for maximizing the utility. Another advantage is introduction of time element, not in the form of several rounds, but longer periods of time.

When exploring the system, we are looking for an optimal, or balanced, state. This is called evolutionary stable balance, which occurs when these two conditions are met (Cvoj, 2011):

- Pay-off of the player with optimal strategy is higher than that of the player with mutation.
- Pay-off of the player with optimal strategy in a combat with a mutant is higher than in the case of a mutant fighting with another mutant.

However, this evolutionary approach has also several limitations consisting mainly in hyper intelligent players with lack of dynamics (Naini *et al.*, 2011). Another limitation is a focus on phenotypes representing individual strategies, which prevent cross-breeding (Karlin & Lessard, 1986). Nevertheless, this limitation is not of practical relevance for the application.

Research methodology

In the first phase, the matrix will be analysed from the point of view of the game meaning and its limit parameters. The limit parameters will be set taking into account the economic reality. Only the states of the matrix where the result is not obvious will be examined. For example, if the VMI model operation costs for the supplier are N , the operation costs of a conventional system must be higher, that is, in the interval $\langle N; \infty \rangle$. If we consider the costs related to the implementation of the model, such costs must meet the requirement of the payback period length that would be acceptable for the market. For the purposes of the contribution, we will work with a 5-year period.

The data will be based on the first experiment described in the article entitled *An evolutionary game theory approach to modelling VMI policies* (Torres & García-Díaz, 2018). Subsequently, sensitivity analysis will be carried out. This will be based on the fixation of all model parameters except for one that will be set after a specific step from one limit state to another.

Since in the case of more complex games the Dynamo software calculations (Sandholm *et al.*, 2012) are demanding for the available computing technology, a new software capable of calculating evolutionary focus and their stability will be created. The newly created software will be verified by carrying out 10 experimental measurements, when the same parameters will be set both in the newly created software and in the Dynamo software. If the same results are obtained in all cases, the new software will be considered reliable and will be applied for the purposes of the sensitivity analysis.

Calculation of focus and evolutionary analysis

Table 1 shows pay-off matrix based on the work of Torres (2014) and is partly regulated for the customer and manufacturer (individuals). Each individual follows exactly one of two strategies described below:

- Buyer – introduce VMI or no introduce VMI;
- Manufacturer – introduce VMI or no introduce VMI.

Pay-off is the cost of supply. Parameter in the table means M = manufacturer, B = buyer, VMI = introduce VMI. It results from the table that the costs for both subjects will be lowest if the VMI is introduced. If neither of them introduced the VMI, they would be at the original optimized total costs (TC). If one of the companies violates the agreement during the introduction of the system, the players' original costs will be increased by the p parameter for the company that has violated the agreement, and the m or n parameter for the other company. The p parameter can be seen as a goodwill loss and penalty, if agreed and m or n parameter represents the investment costs invested in an unsuccessful project.

The individual players will contemplate introduction of the system if:

$$TC_{M-VMI} < TC_M \wedge TC_{B-VMI} < TC_B \quad (5)$$

where:

TC_{M-VMI} – total costs manufacturer with VMI;

TC_M – total costs manufacturer without VMI;

TC_{B-VMI} – total costs buyer with VMI;

TC_B – total costs buyer without VMI.

Moreover, it must hold true that:

$$p_1 > 0 \wedge p_2 > 0 \wedge m > 0 \wedge n > 0 \quad (6)$$

where:

p_1 and p_2 – penalty for the company that has violated the agreement;

m and n – investment costs invested in an unsuccessful project.

It results from the above that:

$$TC_M < TC_M + p_1 \wedge TC_M < TC_M + p_2 \quad (7)$$

$$TC_B < TC_B + m \wedge TC_B < TC_B + n$$

To be able to analyse the given game, replicator dynamic must be known, which must be of the following shape:

$$\frac{d\beta}{dt} = \beta[U_{B-VMI} - \widetilde{U}_B] \quad (8)$$

After substitution and modification:

$$\begin{aligned} \widetilde{U}_B &= \beta U_{B-VMI} + (1 - \beta)U_B \\ \frac{d\beta}{dt} &= \beta[U_{B-VMI} - \beta U_{B-VMI} - (1 - \beta)U_B] \\ \frac{d\beta}{dt} &= \beta[(1 - \beta)U_{B-VMI} - (1 - \beta)U_B] \\ \frac{d\beta}{dt} &= \beta(1 - \beta)[U_{B-VMI} - U_B] \end{aligned} \quad (9)$$

If the individual partial derivations are equal to zero, balance points, stable or unstable, can be obtained. For identifying the stability, Jacobi matrix will be used (Friedman, 1991). The matrix is as follows:

$$J = \begin{bmatrix} \frac{\partial}{\partial \alpha} \left(\frac{d\alpha}{dt} \right) & \frac{\partial}{\partial \beta} \left(\frac{d\alpha}{dt} \right) \\ \frac{\partial}{\partial \alpha} \left(\frac{d\beta}{dt} \right) & \frac{\partial}{\partial \beta} \left(\frac{d\beta}{dt} \right) \end{bmatrix} \quad (10)$$

Results

For the purposes of analysing the impact of pay-offs of the manufacturer when introducing the VMI, in total 33 experiments were carried out, where the payoff of the manufacturer was gradually changed from the value of — 2 614 to hypothetical 0. For each experiment the value was increased by 100. A shortened version of the table showing the input parameters can be seen below (Table 2). The table also shows the E_5 focus.

It results from the table that the changes in values influence the value of balance point E_5 . However, the influence can be seen only in the case of the alpha parameter, which represents the probability of choosing the given strategy for the manufacturer. The value of the alpha parameter is between 0.75 and 0.1. These are the values for parameters set to -2 614, which represents costs lower only 1 unit than in the situation of not introducing the

VMI, and 0, which represents a hypothetical situation with zero costs. If the values were negative, the value of the alpha parameter could be as low as 0. However, this cannot happen. Figure 1 shows the change dynamics. Although there was always the same step (except in the first situation), the shift change of the alpha parameter decreased continuously.

Figure 2 shows a phase diagram that determines a shift stable game solution. The phase diagram principle would be the same for all situations solved; therefore, it will not be generated in the following chapters. Generally, it can be said that there are 2 stable solutions and a focus, from which the solution tends to diverge to the stable one. The experiments identify how the change in parameter moves the given focus, thus affecting the prediction of the system behaviour.

The smaller the alpha parameter is, the less probable it is that the game achieves balanced stable state $E_0 \{1; 1\}$, that is, introduction of the VMI. The balanced state E_5 (see Figure 2) is unstable.

For the purposes of analysing the impact of pay-off change of the customer with introduced VMI, 21 experiments were performed. Other parameters of the model were fixed at their original value. The initial value was set to -1 999, which is the value lower by one unit than in the situation when the customer would not introduce the VMI. Therefore, the initial condition is met. Subsequently the value of the changed variable is reduced by 100 units for each step, except in the first step, when the value is reduced by 99 units. Reduction is performed until the situation of zero costs of holding inventory for the customer. This situation is only a theoretical state for the purposes of identifying the limit conditions. Figure 3 shows the results of the experiments. It results from the graph that the change in the parameter affects mainly the beta parameter. This parameter gradually decreases from 0.71 to the theoretical situation 0.19. If we considered the theoretical situation of infinitely high negative costs (pay-off value would be positive), the value 0 for the beta parameter would be achieved. This, however, cannot happen in reality. If more extreme values for the E_5 results, lower beta parameter values are achieved, there is a situation when the evolutionary stability tends to reach the balanced state E_1 , that is, the strategy of introducing the VMI.

There may be a situation when the manufacturer changes the decision to introduce the VMI system. In such a case, it is necessary to analyse the impact of pay-off change of the manufacturer in the case of not introducing the system. For the manufacturer, this means a loss of the goodwill associated with his brand. The loss is expressed by means of the p_1 parameter added to the standard costs of the model. Adding the parameter has a long term nature, since the reputation of the company can be affected for a long-

er period of time. Also, the sales can fall and the manufacturer can lose the customers. In the model this parameter is gradually lowered by 100 points, except for the first step, where the initial conditions were set to the value — 2 616, that is 1 point higher than the regular costs are. Another exception was the last experiment when the value was reduced from -8 700 to — 70 000. This value was chosen so that the result of the alpha parameter achieved its extreme, which is the 0 value, with an accuracy of 2 decimal places. The input parameters calculations are shown in Figure 4.

It follows from the figure that the additional losses related to breach of the agreement leads to an increase in the alpha value for the focus E_5 . The further the focus from the E_1 point [1; 1], the more likely the system is to converge into balanced solution. In other words, the higher is the loss for the manufacturer, the more likely he is to observe the agreement. From another point of view, it can be stated that in the case of a low goodwill loss it can happen that the system will converge into evolutionary stable balance E_4 [0.0], which represents a state when both parties decide not to introduce the VMI system. The costs will thus be higher in the long run and the system will not be pareto-optimal (Heissler *et al.*, 2010).

If the manufacturer breaches the agreement during the realization of the project, the customer loses the capital invested. The loss depends on the contract terms, and it is possible that the parties will come to agreement or a possible compensation of the loss is contractually guaranteed. The degree of recoverability and quantification of such loss can be a subject of negotiations, discussions and analyses. In our case, this is not taken into account. In total, 33 experiments are carried out. In these experiments the value of other pay-off matrix parameters is fixed. The parameter related to the investment loss is primarily set to 0, which corresponds to the costs for the customer in the case of not introducing the VMI. The results are shown in Figure 5. Even in this case the additional costs cannot be reduced, since they can be higher than the costs of holding inventory. On the other hand, each company considers the payback time when entering a project. If the investment costs were significantly higher, this parameter could not be of a reasonable value and thus the customer would not be willing to realize the project. From a different point of view, both partners can use various subsidies for the VMI implementation, and in the case of breaching the agreement the costs related to the breach of the conditions for granting a subsidy can be really high. In view of the above, a realistic maximum was estimated to the value of -4 000 (payback period in such a case would be approximately 5.5 years). The limit values were set for identifying the approximation of possible solutions. The limit estimated on the basis of the deduction is marked in the figure with different tag and colour.

The situation described in the two aforementioned cases (the manufacturer withdraws from the contract) can occur if it is the customer who withdraws from the contract. In such a case, the manufacturer loses the investments he made in the project. There can also be discussions on recoverability, contracting, and overall amount of acceptable investment that can occur in reality. Deductions in this area have the same logic as in the previous case; therefore, the value of maximum investment was reduced to approx. 1070 units with the overall pay-off value of 3500 in the matrix. In such a case, the payback time is approx. 5.5 years. In total, 33 experiments were carried out, when the value of the changed parameter (the manufacturer's investment loss) was increased by 100 points. The dynamics of the parameters development for E_5 is shown in Figure 6. The extreme values were entered in order to determine the maximum values possible. The realistic values are marked in different colour and tag.

Similarly, the customer can also withdraw from the agreement. Even in this case they lose a certain goodwill value for not realizing the agreed form of the VMI. The size of loss is related to the contractual terms and other parameters of the model. There is no upper limit for the loss, since not realizing can result in the loss of business opportunities that can be higher than the costs of holding the inventory. Figure 7 shows the experiments input values. It follows from the figure that the beta parameter decreases continuously with slowing dynamics (the same change of the step means the smaller change of the beta parameter). The beta parameter value approximates to 0, which can be under certain conditions achieved in reality. This is, however, an extreme case. Due to the slowing dynamics it can be assumed that the parameter value will be about 0.1.

If the VMI is not introduced in production, the manufacturer's costs will be given by the relations defined above (the case of not introducing the VMI in production). In such a case, the costs should have the following parameters, so that the game would make sense:

$$TC_{M-VMI} < TC_M \wedge TC_{B-VMI} < TC_B \quad (11)$$

In other words, the costs of holding inventory should be higher than in the situation when the VMI is introduced. Below (Figure 8) an experiment is shown where the costs of holding inventory parameter changes, while the other parameters are fixed. In addition, the E_5 is calculated. The costs value is set to -2 428 for the first step. The loss is increased to -2 500 for the second step; in the following steps the loss is increased each time by 100 points. Unlike the previous cases, only 7 experiments were carried out. This is due to the fact that after exceeding -2 915 the alpha parameter was out-

side the $\langle 0; 1 \rangle$ interval. In the real situation, there can be a stage when the costs of holding inventory are significantly higher than the costs of holding inventory with the introduced VMI. It results from the above that the E_5 focus may not exist. This does not represent a problem in real situation, since in such a case, stability is in E_1 .

The last possible change of the payoff matrix parameter is the change of pay-off related to not introducing the VMI for the customer (Figure 9). Even in the case, the following condition must be met:

$$TC_{B-VMI} < TC_B \quad (12)$$

The initial values are set from the value -1800. Subsequently the loss is increased by 100 points for each step. Within these experiments, in total 8 calculations were carried out, where the E_5 focus in the last stage was outside the probability limits $\langle 0; 1 \rangle$.

Discussion and limitations of study

It follows from the analyses above that the parameters in the positions A_{11} , A_{12} , A_{21} have a slowing dynamics with respect to the parameters of the resulting stability solution E_5 . In other words, the same change of the pay-off function value results in smaller change of the final shift of the E_5 focus. The E_5 focus is significant from the point of view of the evolutionary stability of the game. E_5 is unstable as such (saddle point), but its position divides the game into 2 parts that converge to stable solution E_1 (introduction of the VMI) and E_4 . For these reasons, its position is a key one and was subject of the aforementioned experiments.

The parameters in the A_{22} position showed a different trend, that is, an accelerating shift dynamics. With the same change of the A_{22} parameters, bigger changes of shift of the E_5 occurred. Even with the assumption of basic formulas, without which the game would not make a logical economic sense, it would be possible to set the game parameters so that the E_5 solution would not be in the interval $\langle 1; 1 \rangle$. Analyzing these states, the assumption that the evolutionary stable balance would converge to the E_1 , that is, introduction of the VMI, was confirmed.

The contribution analyzes the introduction of the model in terms of limits within a payoff matrix. The model is thus generally described from the point of view of the evolutionary game theory regardless the input parameters. The advantage of this approach is the fact that it includes all expressed and non-expressed parameters that affect costs. A disadvantage, however,

is the fact that the contribution does not explore the specific factors determined in the sample costs. This can be a subject of further research. This could make the model even more successful and unique.

Conclusions

The results obtained can be used for practical management, so that the managers are able to identify what the actual costs are and what the probability of introducing the system is. At the same time, they can identify the parameters that can be influenced by them and observe their impact on the shift of the system introduction probability.

The above mentioned analyses complement the original results of Torrez *et al.* (2014), and Torrez and Garcia-Diaz (2018) and extended significantly this model. As a result, a complex analysis of the aforementioned VMI model was created. This contribution was elaborated based on the summary of the outputs of Stehel's thesis (2018). The limitation of current approach is the fact that it not includes all expressed and non-expressed parameters that affect costs. We can explore the specific factors determined in the sample costs.

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Annex

Table 1. Pay-off matrix for VMI

	VMI buyer		Buyer	
VMI manufacturer	TC_{M-VMI}	TC_{B-VMI}	$TC_M + m$	$TC_B + p_2$
Manufacturer	$TC_M + p_1$	$TC_B + n$	TC_M	TC_B

Source: Torres (2014).

Table 2. Changes in pay-offs of manufacturer with introduced VMI

	A11		A21		A12		A22		E ₅	
Manufacturer r VMI	Customer VMI	Manufacturer	Customer	Manufacturer	Customer	Manufacturer	Customer	Alfa	Beta	
- 2 614	-1 799	-2 715	-2 500	-2 915	-2 200	-2 615	-2 000	0.75	0.55	
- 2 600	-1 799	-2 715	-2 500	-2 915	-2 200	-2 615	-2 000	0.72	0.55	
- 2 500	-1 799	-2 715	-2 500	-2 915	-2 200	-2 615	-2 000	0.58	0.55	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
0	-1 799	-2 715	-2 500	-2 915	-2 200	-2 615	-2 000	0.10	0.55	

Figure 1. Impact of changing TC_{M-VMI} to E_5

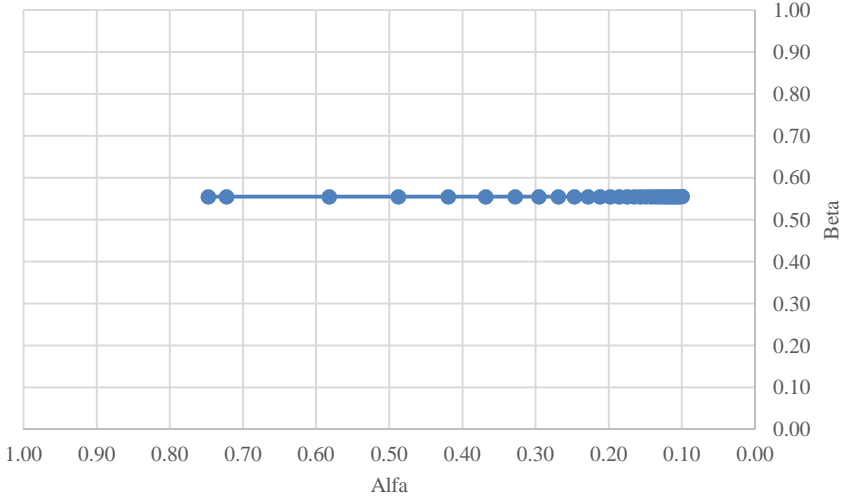


Figure 2. Phase diagram

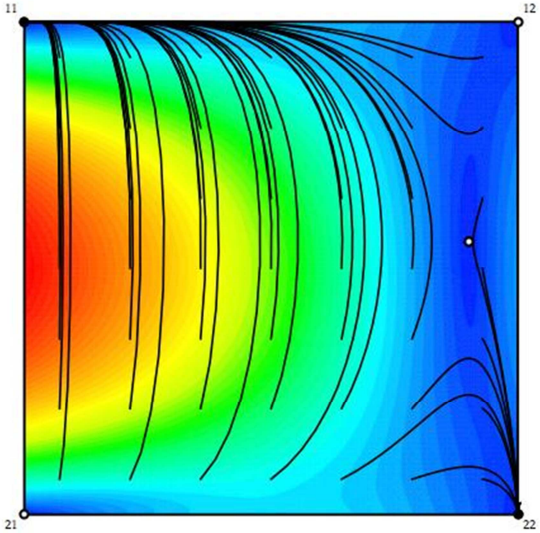


Figure 3. Change of pay-off value for customer with introduced VMI

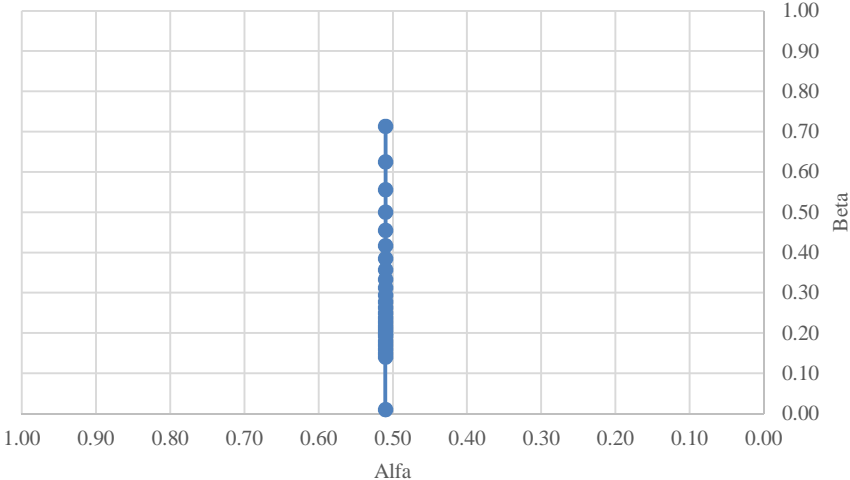


Figure 4. Breach of agreement from the side of manufacturer

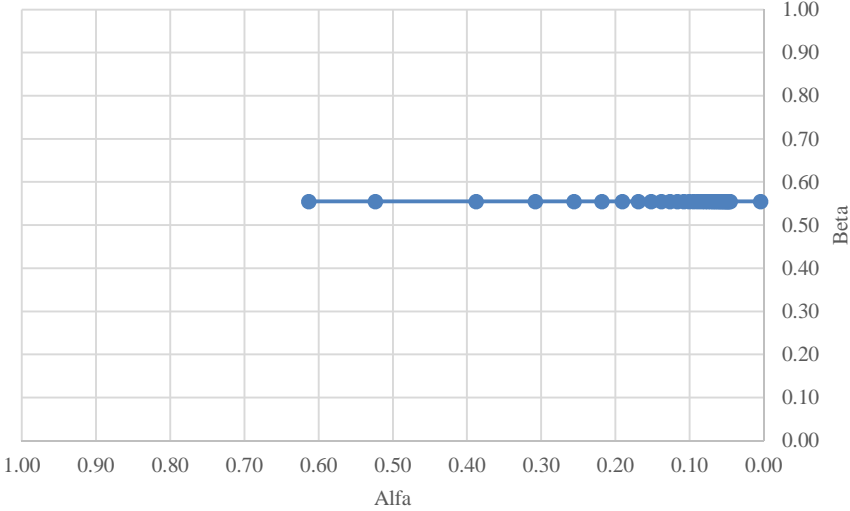


Figure 5. Additional costs for customer in case of breaching the agreement from the side of manufacturer

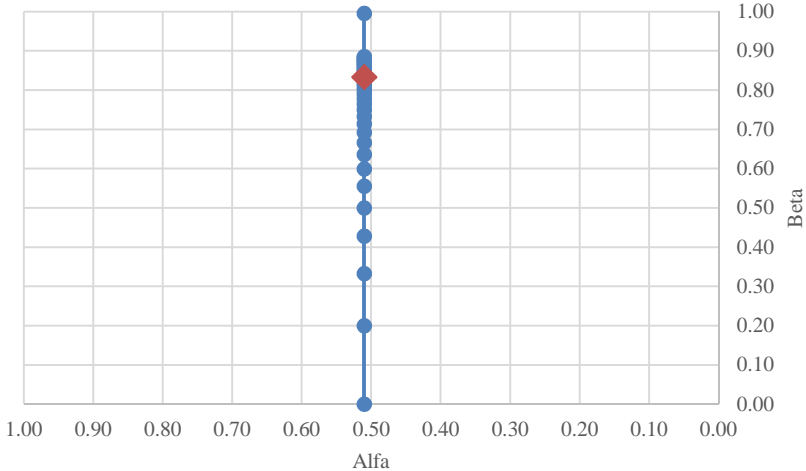


Figure 6. Development of alpha and beta parameters based on investment loss for manufacturer

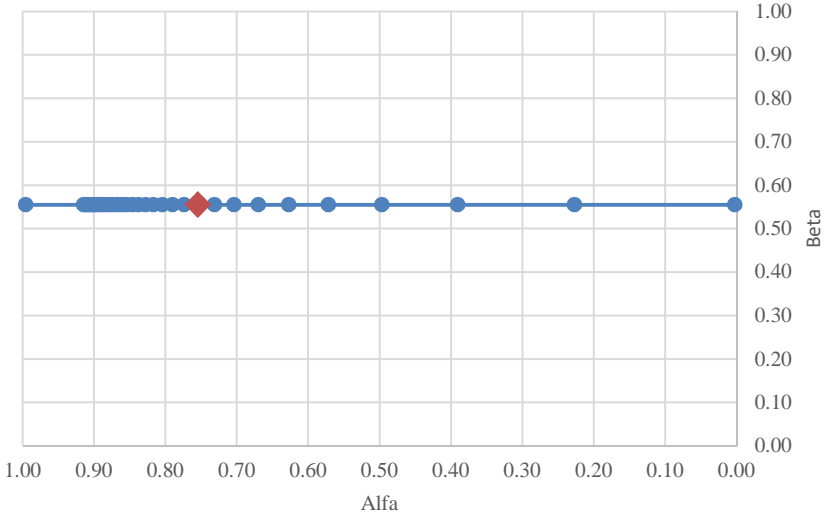


Figure 7. Customer's withdrawal from agreement

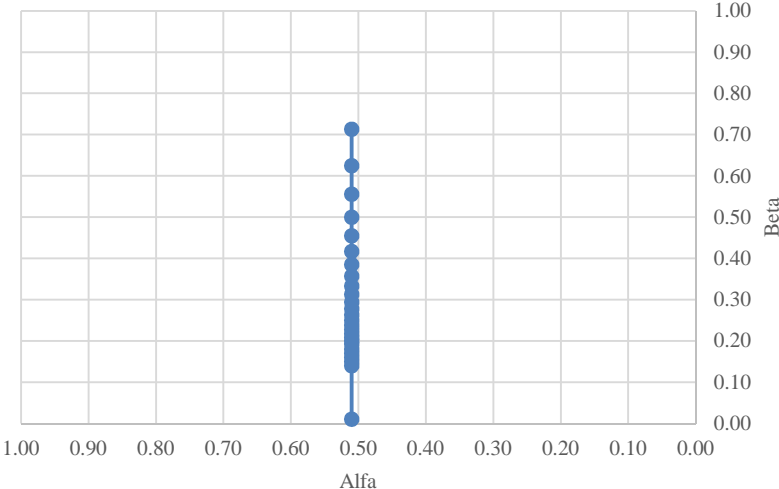


Figure 8. Change of pay-off function for manufacturer without VMI

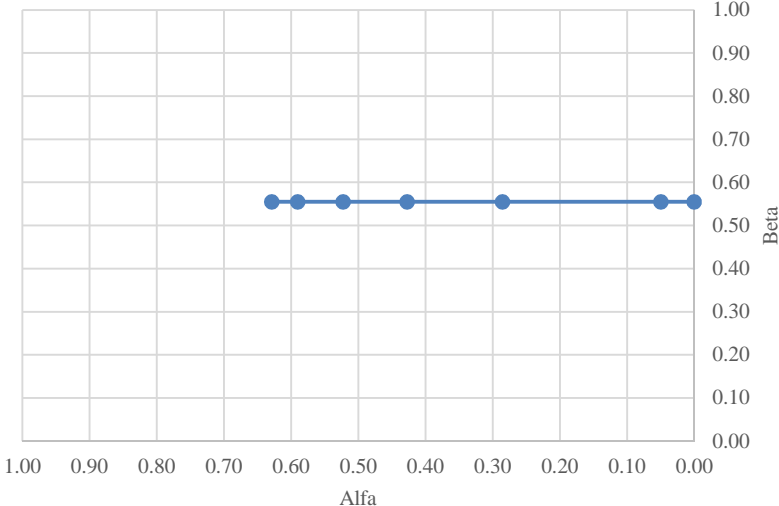


Figure 9. Change of E_5 for changing pay-off for customer in the case of not introducing the VMI

