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Hysteresis and stochastic convergence in Eurozone unemployment rates: evidence from panel unit roots with smooth breaks and asymmetric dynamics

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Abstract

Research background: Studying the dynamic characteristics of unemployment rate is crucial for both economic theory and macroeconomic policies. Despite numerous research, the empirical evidence about stochastic behaviour of the unemployment rate remains disputable. It has been widely agreed that most economic variables, including unemployment rates, are characterized by both structural breaks and nonlinearities. However, a little work is done to examine both features simultaneously.

Purpose of the article: In this paper, we analyse the stationarity properties of unemployment rates of Euro area member countries. Also, we aim to test stochastic convergence of unemploy-

ment rates among member countries. Our empirical procedures explicitly allow for simultaneous gradual breaks and nonlinearities in the series.
Methods: This paper develops a new unit root test procedure for panel data, allowing for both gradual structural breaks and asymmetric adjustment towards equilibrium. We carry out Monte Carlo simulations to examine small sample performance of the proposed test procedure and compare it to the existing test procedures. We apply the newly proposed test to examine the stochastic properties of the unemployment rates of Euro-member countries as well as relative unemployment rates vis-à-vis the Eurozone unemployment rate.
Findings & value added: We find that the newly developed test procedure outperforms existing tests in highly nonlinear settings. Also, these tests reject the null hypothesis of unit root in more cases when compared to the existing tests. We find stationarity in the series only after allowing for structural breaks in the data generating process. Allowing for nonlinear and asymmetric adjustment in addition to gradual breaks provides evidence of stationarity in more cases. Furthermore, our results suggest that relative unemployment rate series are stationary, providing evidence in favour of stochastic convergence in unemployment rates. Overall, our results imply a limited room for coordinated economic policy to fight unemployment in the Eurozone.

Introduction

The dynamic behaviour of unemployment rate has been one of the most disputable issues in macroeconomic theory. According to the natural rate of unemployment hypothesis developed by Phelps (1967) and Friedman (1968), natural unemployment rate is determined by real forces and monetary policy may cause only temporary deviations from the natural rate. On the other hand, the hysteresis hypothesis, proposed by Blanchard and Summers (1986), suggests that deviations of the unemployment rate from its equilibrium level may cause a shift in the level of the long-run equilibrium itself, and hence, have permanent effects.

Knowledge of the true nature of the unemployment rate is also crucial for policy authorities as unemployment has been one of the major policy considerations, especially during hard times. If deviations from the natural rate are transitory, monetary and fiscal policies will have no permanent effect on the unemployment. Demand management policies will reduce unemployment only temporarily in the short run, but often at the cost of high inflation. In the long run, however, unemployment will return to its natural level with inflation remaining stable at the increased level. This feature of the unemployment rate to revert to its equilibrium value, therefore, renders such interventionist policies inappropriate and undesirable in the face of huge costs of rising inflation. In this case, the policies should aim at changing the institutions and structure of the goods and labour markets to tackle the chronic unemployment problem. The hysteresis hypothesis, proposed by Blanchard and Summers (1986), on the other hand, has been considered as a rationalization of proactive economic policies. If the hysteresis hypothesis holds, policy authorities can implement demand stimulating policies to prevent rising cyclical unemployment from transforming into higher structural unemployment (Delong & Summers, 2012).

In this regard, examining the stochastic properties of unemployment rates is important especially in the case of the Euro member countries. Adapting common currency not only means forgoing control over monetary policy, but also puts strict restrictions on fiscal policies as well. As the control over domestic economic policy is severely restricted, the Eurozone countries must carefully consider their economic policies as well as policy priorities. Thus, given the restricted room for independent demand management policies, the economic policy authorities of these countries need to know the exact nature of unemployment dynamics to design and implement appropriate policies.

Also, appropriateness of the European Central Bank's intervention policy is highly conditional on the co-movement of economic variables of the member countries. Co-movement or stochastic convergence of economic variables is considered as the most important precondition of currency unions (see, e.g. Kutan & Yigit, 2005). If economic variables of different countries within a currency area do not share common trends, economic policy favourable for one country might be detrimental for the other. Therefore, design and implementation of monetary policy acceptable to all the member countries also require the knowledge of stochastic properties of relative unemployment rates. If the unemployment rates of the member countries are cointegrated, a monetary policy suitable for all member countries can be designed in principle, and consensus on such a policy can be achieved. On the other hand, if the series are not cointegrated, no monetary policy will fit the needs of all countries. For this purpose, in addition to stationarity of individual unemployment rates, we also analyse the stochastic convergence of unemployment rates. Specifically, we also test cointegration of individual countries' unemployment rates with the Eurozone unemployment rate as well (for a thorough discussion of types and tests of convergence, one may refer to Kónya, 2020).

In this paper we aim to provide a new insight into the stochastic properties of unemployment rates for a panel of 19 Eurozone countries. It has long been recognized that most economic variables are characterised by both structural breaks and nonlinearities (see, for example, Neftçi, 1984; Leybourne *et al*., 1998; Kapetanios *et al*., 2003). Existing panel unit root tests, on the other hand, consider either structural breaks (e.g., Im *et al*., 2005; Bai & Carrion-i-Silvestre, 2009; Lee *et al*., 2016; Omay *et al.,* 2018b) or nonlinearities (e.g., Uçar & Omay, 2009; Emirmahmutoglu & Omay, 2014). Taking account of the fact that both structural breaks and nonlinearities might be more appropriate for economic variables, we propose a new panel a new panel unit root test procedure that allows for simultaneous structural change and nonlinear adjustment. To this end, we follow Leybourne *et al* (1998) and Omay *et al.* (2018a, and 2018b) and use the logistic smooth transition (LST) function to model structural breaks. Adjustment towards the gradually changing attractor is modelled using the asymmetric exponential smooth transition (AESTAR) model proposed by Sollis (2009). While the LST model allows for gradual change between regimes, the AESTAR model allows for asymmetric adjustment towards the equilibrium, whereas the speed of adjustment depends both on the size and sign of the deviations from the equilibrium. These models have a very useful property of allowing for gradual transitions, while nesting the abrupt change, no break, and linear models as special cases. The results of the Monte Carlo simulations suggest that the proposed tests have rather satisfactory size and power properties even in small panels.

The rest of the paper is organized as follows. In next section, we provide a brief literature review of the studies on unemployment hysteresis and stochastic convergence. In the third section, we present the new panel unit root testing procedure and provide results of the Monte Carlo simulations. The fourth section applies the newly developed tests to the Euro area panel and presents the results. The fifth section contains a discussion of the results, and sixth section concludes.

Literature review

On theoretical grounds, there are two competing views on stochastic behaviour of the equilibrium unemployment rate. According to the natural rate of unemployment (the "NRU") hypothesis, developed by Phelps (1967) and Friedman (1968), any deviation of the actual unemployment rate from the natural rate is short-lived. Friedman (1968, pp. 8–9) argued that NRU is determined within the Walrasian system of general equilibrium equations, "… provided that the actual structural characteristics of the labour and commodity markets is imbedded in them, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labour availabilities, the cost of mobility, and so on". Removal of market frictions would tend to lower the natural rate of unemployment. On the other hand, demand management policies would cause only a temporary deviation from the NRU. Modigliani and Papademos (1975) refined the NRU concept further and introduced the term "noninflationary rate of unemployment", which later became known as "non-accelerating inflation rate of unemployment" (the "NAIRU"). The

NAIRU is the lowest level of unemployment that can be sustained without causing inflation to rise.

The hysteresis hypothesis proposed by Blanchard and Summers (1986), on the other hand, suggests that deviations from the long-run equilibrium unemployment rate may well persist into the feature by causing a shift in the natural rate of unemployment. As argued by Blanchard and Summers (1986), fundamental asymmetries in the wage-setting process between insiders (those who are currently employed) and outsiders (those who are unemployed and are job seekers) may lead to persistent deviations from the NRU. A shock that causes a change in employment alters the number of insiders in effect. In this case, temporary shocks may have permanent effects on the level of employment if equilibrium wage rates are bargained between employers and insiders. In addition to wage setting behaviours, depreciation of skills (Blanchard, 1991), self-enforcing habit formation (Vendrik, 1993), path dependent stigma effects (Sessions, 1994) and other market imperfections may also lead to high unemployment persistence (for a thorough discussion of this issue, see Blanchard & Katz, 1997; Røed 1997).

Given the importance of unemployment rate for both macroeconomic theory and policymakers, testing empirical validity of the NRU and hysteresis hypotheses has attracted a great deal of attention among researchers. The accumulated empirical evidence, however, is mixed at best. For instance, Blanchard and Summers (1986), Brunello (1990), Mitchell (1993), Røed (1996), among others, provided empirical evidence supporting the hysteresis effects in most of the sample countries. On the other hand, Song and Wu (1998), Camarero and Tamarit (2004), Lee and Chang (2008), among others, reported results in line with the natural rate hypothesis. Røed (1997) provides a detailed survey of the earlier research. For a recent survey, one may refer to O'Shaughnessy (2011), Akdoğan (2017) and Furuoka (2017).

Having been dissatisfied with the contradictory results, the researchers have been searching for alternative econometric methods that fit the theoretical and empirical features of the unemployment series more satisfactorily. One strand of the literature suggested using econometric methods allowing for structural changes in the unemployment rates. In fact, as Friedman (1968) argued, changes in the characteristics of markets may have caused the equilibrium unemployment to shift. Naturally, failure to consider such shifts may result in a rejection of the unit root null in the unemployment series. León-Ledesma and McAdam (2004), Lee and Chang (2008), Furuoka (2017), Jiang *et al*. (2019), Krištić *et al.* (2019) and Omay *et al.* (2021), among others, found that the unemployment series are stationary after allowing for structural breaks, thus providing empirical support for the natural rate hypothesis. On the other hand, even after allowing for structural breaks in the series, Chang (2011) and Çağlayan Akay *et al.* (2021) failed to reject the null hypothesis of unit root in most of the OECD countries and transition countries, respectively, consistent with the hysteresis hypothesis.

Another strand of the literature pointed to the possibility of nonlinear and/or asymmetric dynamics. It has long been recognized that dynamics of many economic variables might be inherently nonlinear. For example, when discussing business cycle dynamics, Keynes (1936) pointed out that recessions usually take place suddenly and violently, whereas recovery happens gradually. By the same token, Neftçi (1984) found that unemployment rises suddenly, but falls slowly over the course of the business cycle, proving that unemployment displays asymmetric dynamics. Following the seminal contribution of Neftçi (1984); Rothman (1991), Koop and Potter (1999), Caner and Hansen (2001), Skalin and Terasvirta (2002), Akdoğan (2017), among others, have provided strong evidence for nonlinearities in the unemployment series. Using tests allowing for possible nonlinearities Caner and Hansen (2001), Franchi and Ordonez (2008), Lee (2010), Meng *et al*. (2017) provided empirical support for the NRU hypothesis. Yılancı (2008) and Perez-Alonso and Di Sanzo (2011), Lukianenko *et al.* (2020), Yaya *et al.* (2021) on the other hand, presented some evidence supporting the hysteresis proposition.

Apart from stochastic properties of individual series, convergence of unemployment rates across countries has also attracted the interest of researchers (see Krištić *et al*., 2019; Kónya, 2020). Convergence in economics refers to the property of catching-up the leaders by the initial laggard countries, especially in terms of per capita income or productivity. There are three widely used concepts of convergence in economics. The sigma-convergence (or σ-convergence) implies declining cross-sectional dispersion of the variable of interest over time. Notice that this concept relates to cross-sectional distribution of the variable. Therefore, the catch-up hypothesis is usually conceptualized and tested by the stochastic convergence or the β-convergence. As further elaborated by Kónya (2020), stochastic convergence occurs when shocks to the deviations of the variable of interest from time-varying benchmark are temporary. Thus, the stochastic convergence implies that a linear combination of variables is a stationary process. The β-convergence, on the other hand, places a restriction on the coefficients of the linear regression of the difference series on a constant and time trend. Estrada *et al*. (2013) find that while the first nine years of the currency union witnessed strong convergence in unemployment rates across the Euro area countries, this process was interrupted and reversed during the

2008 financial crisis. Beyer and Stemmer (2016) also find that the regional unemployment in Europe exhibited convergence till 2007, whereas polarized after the financial crisis, thus providing evidence for the existence of structural breaks. Krištić *et al.* (2019), Carrera and Rodriguez (2009), and Kónya (2020) found strong evidence supporting stochastic convergence of unemployment rates across Eurozone and EU countries, after allowing for structural breaks.

Research methodology

Both the NRU and hysteresis hypotheses can be tested by examining the stochastic properties of the unemployment rate series. Stationarity of the series implies that deviations from the long-run equilibrium level are temporary. On the other hand, if the series follow a nonstationary unit root process, deviations do not die out with elapse of time, suggesting that shocks to equilibrium level are persistent. Thus, stationarity of unemployment rates supports the NRU hypothesis, whereas non-stationary unemployment rates verify empirical fulfilment of the hysteresis hypothesis. Similarly, stochastic convergence of series is usually tested by stationarity of a linear combination of the series, whereas stationarity implies convergence of the series (see Krištić *et al*., 2019; Kónya, 2020).

Considering the fact that unemployment rate series may be subject to structural breaks and exhibit nonlinearities, we propose a new panel unit root test procedure that allows for both structural change and nonlinearity in unemployment rates. Existing panel unit root tests consider either structural breaks (e.g., Im *et al*., 2005; Bai & Carrion-i-Silvestre, 2009; Lee *et al*., 2016; Omay *et al.,* 2018b) or nonlinearities (e.g., Uçar & Omay, 2009; Emirmahmutoglu & Omay, 2014). The test procedure proposed in this paper, however, not only allows for a gradual structural change to take place between regimes, but also accounts for an asymmetric adjustment towards the nonlinear attractor. To this end, we follow Leybourne *et al* (1998) and Omay *et al*. (2018a, 2018b) and use the logistic smooth transition (LST) function to model structural breaks. Adjustment towards the gradually changing attractor is modelled using the asymmetric exponential smooth transition (AESTAR) model proposed by Sollis (2009). While the LST model allows for gradual change between regimes, the AESTAR model allows for asymmetric adjustment towards the equilibrium where the speed of adjustment depends both on the size and on the sign of the deviations from the equilibrium. These models have the very nice property that they allow for gradual transitions, while nesting the abrupt change, no break, and linear models as special cases. The results of the Monte Carlo simulations suggest that the proposed tests have rather satisfactory size and power properties even in small panels.

Modelling gradual breaks and nonlinear adjustment

We consider the following data generating process (DGP):

$$y_{it} = \phi_{it} + u_{it} \tag{1}$$

for $i = 1,2, \dots, N$ cross-section units and $t = 1,2, \dots, T$ time periods. Here, ϕ_{it} is the nonlinear trend function (or "the attractor"), and u_{it} are the deviations from this trend. The nonlinear trend function ϕ_{it} is modelled using the logistic smooth transition (LST) function:

Model A $\quad \phi_{it} = \alpha_{1i} + \alpha_{2i} S_{it}(\gamma_i, \tau_i)$ (2a)

Model B $\quad \phi_{it} = \alpha_{1i} + \beta_{1i} t + \alpha_{2i} S_{it}(\gamma_i, \tau_i)$ (2b)

Model C $\quad \phi_{it} = \alpha_{1i} + \beta_{1i} t + \alpha_{2i} S_{it}(\gamma_i, \tau_i) + \beta_{2i} t S_{it}(\gamma_i, \tau_i)$ (2c)

where $S_{it}(\gamma_i, \tau_i)$ is the logistic smooth transition function over sample of T:

$$S_{it}(\gamma_i, \tau_i) = [1 + \exp\{-\gamma_i(t - \tau_i T)\}]^{-1}, \gamma_i > 0 \tag{3}$$

The transition function $S_{it}(\gamma_i, \tau_i)$ is bounded between zero and one, and governs the transition between the regimes. The parameter γ_i controls the pace of the transition whereas the parameter τ_i determines the timing of the transition. Notice that when $\gamma_i = 0$, the transition function collapses to a constant, i.e. $S_{it}(\gamma_i = 0, \tau_i) = 1/2$ for $\forall\ \tau_i$. At the other extreme, when $\gamma_i \to +\infty$, $S_{it}(\gamma_i, \tau_i) = 0$ for $t < \gamma_i T$ and $S_{it}(\gamma_i, \tau_i) = 1$ for $t > \gamma_i T$, and $S_{it}(\gamma_i, \tau_i) = 1/2$ for $t = \tau_i T$. Thus, the LST model given in eq. (3) nests the no-break and instantaneous break models as a special case. Model A given in eq. (2a) does not include a time trend, thus allowing for a break in the mean of non-trending series whose mean changes from initial value α_{1i} to the final value $\alpha_{1i} + \alpha_{2i}$. While Model B in eq. (2b) allows for a trend in the series, structural change is restricted to the mean only. Finally, Model C in eq. (2c) allows for a break both in the mean and trend of the series (see also Leybourne *et al.*, 1998; Omay *et al.,* 2018b).

We model the adjustment of the deviations u_{it} from the nonlinear attractor ϕ_{it} using the asymmetric exponential smooth transition (AESTAR) model proposed by Sollis (2009):

$$\Delta u_{it} = G_{it}(\theta_{1i}, u_{it-1})\{F_{it}(\theta_{2i}, u_{it-1})\rho_{1i} + \left(1 - F_{it}(\theta_{2i}, u_{it-1})\right)\rho_{2i}\}u_{it-1} + \epsilon_{it} \quad (4)$$

with

$$G_{it}(\theta_{1i}, u_{it-1}) = 1 - \exp\left(-\theta_{1i}\left(u_{it-1}^2\right)\right), \theta_{1i} > 0 \quad (5)$$

$$F_{it}(\theta_{2i}, u_{it-1}) = [1 + \exp(-\theta_{2i}u_{it-1})]^{-1}, \theta_{2i} > 0 \quad (6)$$

where we initially assume that $E(\epsilon_{it}) = 0$, $E\left(\epsilon_{it}^2\right) = \sigma_i^2$, and $E\left(\epsilon_{it}, \epsilon_{js}\right) = 0$ for $i \neq j$ and $t \neq s$. Later, we will relax the assumption $E\left(\epsilon_{it}, \epsilon_{js}\right) = 0$ to allow for cross-sectional dependence among the panel members and serial correlation.

This specification of the deviations u_{it} given in eq. (4)-(6) can accommodate a wider range of dynamic adjustment towards the gradually changing attractor ϕ_{it}. In fact, notice that the exponential transition function $G_{it}(\theta_{1i}, u_{it-1})$ is a symmetrically U-shaped function bounded between zero and one. As the deviations u_{it-1} enter the exponential function $G_{it}(\theta_{1i}, u_{it-1})$ as a quadratic term, the regimes associated with $G_{it}(\theta_{1i}, u_{it-1})$ function are determined by the size of the deviations, irrespective of the sign. On the other hand, the regimes associated with the logistic function $F_{it}(\theta_{2i}, u_{it-1})$ are determined by the sign of deviations given that u_{it-1} is a zero-mean variable. Thus, adjustment of the deviations towards the equilibrium depends both on the sign and size of the deviations.

To see the nature of the adjustment, first consider the case where deviations u_{it-1} move from zero to minus infinity. In this case, the logistic function $F_{it}(\theta_{2i}, u_{it-1}) \to 0$, and hence, eq. (4) collapses to the ESTAR model considered by Kapetanios *et al*. (2003) for time series:
$\Delta u_{it} = \rho_{2i}G_{it}(\theta_{1i}, u_{it-1})u_{it-1} + \epsilon_{it}$

This model implies transition from the inner regime

$$\Delta u_{it} = \epsilon_{it} \quad (7)$$

to the outer regime

$$\Delta u_{it} = \rho_{2i}u_{it-1} + \epsilon_{it} \quad (8)$$

since the exponential function $G_{it}(\theta_{1i}, u_{it-1})$ moves from zero to one as $u_{it-1} \to -\infty$. Notice that the speed of the transition between these two regimes depends on the value of the parameter θ_{1i} where small values imply rather gradual transition between regimes.

On the other hand, if the deviations go from zero to positive infinity, then the logistic transition function $F_{it}(\theta_{2i}, u_{it-1}) \to 1$ as $u_{it-1} \to +\infty$, and eq. (4) reduces to

$$\Delta u_{it} = \rho_{1i} G_{it}(\theta_{1i}, u_{it-1}) u_{it-1} + \epsilon_{it}$$

implying transition from the inner regime

$$\Delta u_{it} = \epsilon_{it}$$

to the outer regime

$$\Delta u_{it} = \rho_{1i} u_{it-1} + \epsilon_{it} \tag{9}$$

as the exponential function $G_{it}(\theta_{1i}, u_{it-1})$ also moves from zero to one when $u_{it-1} \to +\infty$.

Global stationarity of the AESTAR process given above requires $\theta_{1i} > 0$, $\rho_{1i} < 0, \rho_{2i} < 0$ (see Sollis, 2009). As the logistic transition function $F_{it}(\theta_{2i}, u_{it-1})$ given in equation (6) is bounded between zero and one, the coefficient on lagged deviation u_{it-1} in eq. (5) is a weighted average of two coefficients, ρ_{1i} and ρ_{2i}. Therefore, it follows that the deviations from equilibrium u_{it} is geometrically ergodic, and hence, asymptotically stationary if $\theta_{1i} > 0$, $\rho_{1i} < 0, \rho_{2i} < 0$. When $\theta_{1i} = 0$, the exponential function given in eq. (5) will also be equal to zero, $G_{it}(\theta_{1i}, u_{it-1}) = 0$, and hence, eq. (4) will collapse to $\Delta u_{it} = \epsilon_{it}$, implying that deviations from the equilibrium follow a unit root process. On the other hand, if $\theta_{1i} > 0$, u_{it} will follow a nonlinear, but stationary, process provided that $-2 < [F_{it}\rho_{1i} + (1 - F_{it})\rho_{2i}] < 0$, which is assumed to hold. For formal proof, see Kapetanios *et al.* (2003).

Notice that when $\rho_{1i} = \rho_{2i}$ or $\theta_{2i} = 0$, the model gives the symmetric ESTAR adjustment towards equilibrium as proposed by Kapetanios *et al.* (2003). When $\rho_{1i} \neq \rho_{2i}$ and $\theta_{2i} > 0$, on the other hand, one will obtain an asymmetric adjustment towards the nonlinear attractor where the speed of adjustment towards equilibrium depends not only on the size, but on the sign of the deviation as well. Notice that this specification also nests a linear adjustment towards equilibrium. When $\theta_{1i} \to +\infty$ and $\rho_{1i} = \rho_{2i} = \rho_i$ one obtains

$$\Delta u_{it} = \rho_i u_{it-1} + \epsilon_{it} \tag{10}$$

since $G_{it}(\theta_{1i}, u_{it-1}) = 1$ as $\theta_{1i} \to +\infty$. Thus, this specification nests linear panel unit root test of Im *et al.* (2003) (with $\gamma_i = 0$, $\theta_{1i} \to +\infty$ and $\theta_{2i} = 0$), panel gradual break test of Omay *et al.* (2018b) (with $\gamma_i > 0$, $\theta_{1i} \to +\infty$ and $\theta_{2i} = 0$ or $\rho_{1i} = \rho_{2i}$), and nonlinear panel unit root test of Ucar and Omay (2009) (with $\gamma_i = 0$, $0 < \theta_{1i} < +\infty$ and $\theta_{2i} = 0$).

Notice that, in our context, size asymmetry (i.e., the case where the nonlinearity stems from the size of the deviations from the equilibrium) can be justified by the presence of hiring and firing costs. In fact, if the costs of hiring (or firing) are sufficiently high, then the firms will be reluctant to incur these costs and adjust their employment towards the desired level in the face of demand shocks. If the size of the demand shocks is sufficiently high to cover the hiring/firing costs, however, the firms will incur these costs and adjust the actual employment towards the desired levels. Moreover, if the hiring and firing costs are comparable, the speed of adjustment of disequilibrium towards the desired level will depend only on the size of the shock, but not the sign. If the hiring costs and the firing costs are quite different from each other, on the other hand, the speed of adjustment will depend on the sign of the demand shocks as well. In addition, employment promoting policies and incentives will also cause asymmetric adjustment to occur in employment. Thus, on theoretical grounds, simultaneous sign and size asymmetry in the adjustment of unemployment is more appropriate than either of nonlinear or linear models.

Testing the null hypothesis of unit root

The null hypothesis of unit root in deviations u_{it} against the globally stationary AESTAR can be tested by:

$$H_0: \theta_{1i} = 0 \text{ for } \forall\, i \tag{11}$$

against the alternative:

$$H_1: \theta_{1i} > 0 \text{ for some } i \tag{12}$$

However, the presence of unidentified nuisance parameter under the null makes direct testing impossible. In fact, notice that the parameters θ_{2i}, ρ_{1i} and ρ_{2i} are not identified under the null hypothesis. Previous researchers circumvented this problem by replacing the transition functions by an appropriate Taylor series approximation (see, e.g., Luukkonen *et al.,* 1988;

Kapetanios *et al.,* 2003). Therefore, following the literature, we replace both transition functions given in eq. (5) and (6), and obtain the following regression model:

$$\Delta u_{it} = \delta_{1i} u_{it-1}^3 + \delta_{2i} u_{it-1}^4 + \vartheta_{it} \tag{13}$$

where u_{it} is the deviations from the nonlinear trend function, $\delta_{1i} = \theta_{1i}\rho_{2i}$, $\delta_{2i} = \frac{1}{4}\theta_{1i}\theta_{2i}(\rho_{1i} - \rho_{2i})$ and $\vartheta_{it} = \epsilon_{it} + R_{it}$. Here, ϵ_{it} is the disturbance term and R_{it} is the Taylor remainder. After this approximation, testing the null hypothesis $H_0\colon \theta_{1i} = 0$ becomes equivalent to testing the following null hypothesis in eq. (13):

$$H_0\colon \delta_{1i} = \delta_{2i} = 0 \tag{14}$$

Now, we relax the assumption $E\left(\epsilon_{it}, \epsilon_{js}\right) = 0$ for $i \neq j$ and $t \neq s$ in eq. (4) to allow for serial and cross-sectional correlations. Specifically, we adopt the following single-factor structure proposed by Pesaran (2007) for the disturbances ϵ_{it}:

$$\epsilon_{it} = \varrho_i \epsilon_{it-1} + \nu_{it}, |\, \varrho_i| < 1 \tag{15}$$

$$\nu_{it} = \lambda_i f_t + \xi_{it} \tag{16}$$

$$\text{with } \xi_{it} \sim iid.\, N\left(0, \sigma_i^2\right) \text{ and } f_t \sim N\left(0, \sigma_f^2\right)$$

where f_t is the unobserved common factor, λ_i are the factor loadings, and ξ_{it} are idiosyncratic errors (see Omay *et al.*, 2018b for a detailed discussion of this structure).

Following Omay *et al.* (2018b), we use two alternative methods to deal with the presence of cross-sectional dependence (CSD) among the disturbance terms ϵ_{it}. First, we use the Sieve bootstrap re-sampling procedure in the following regression equation to derive empirical distribution of the test statistics:

$$\Delta u_{it} = \delta_{1i} u_{it-1}^3 + \delta_{2i} u_{it-1}^4 + \textstyle\sum_{j=1}^{p} \varphi_{ij} \Delta\, u_{it-j} + e_{it} \tag{17}$$

Notice that, under the assumption that the errors are not correlated across cross-sectional units but are serially correlated (i.e., $\lambda_i = 0$ for $\forall\, i$ in eq. (16)), one also obtains the augmented test eq. (17). Thus, if disturbances do not suffer from CSD, one may still use the test eq. (17).

Second, we use the common correlated effects (CCE) estimator proposed by Pesaran (2006). Pesaran has shown that the unobserved common factor f_t can be approximated by the cross-sectional averages of the regressand and regressors. This approximation yields the following augmented test regression:

$$\Delta u_{it} = \delta_{1i} u_{it-1}^3 + \delta_{2i} u_{it-1}^4 + \kappa_{1i} \bar{u}_{t-1}^3 + \kappa_{2i} \bar{u}_{t-1}^4 + \sum_{j=1}^{p} \varphi_{ij} \Delta u_{it-j} + \sum_{j=0}^{p} \psi_{ij} \Delta \bar{u}_{it-j} + e_{it} \tag{18}$$

for the serially and cross-sectionally correlated errors.

The unit root null hypothesis can now be tested using the conventional F statistic for $\delta_{1i} = \delta_{2i}$ in regression eq. (13), eq. (17) or eq. (18) depending on the nature of cross-sectional and/or serial correlation of residuals. Denoting the individual test statistics as $F_{i,\alpha}$ for model A (eq. 2a), $F_{i,\alpha(\beta)}$ for model B (eq. 2b), and $F_{i,\alpha\beta}$ for model C (eq. 2c), one can compute the mean group statistics as (See also Emirmahmutoglu & Omay, 2014):

$$\bar{F}_{\alpha} = N^{-1} \textstyle\sum_{i=1}^{N} F_{i,\alpha} \tag{19a}$$

$$\bar{F}_{\alpha(\beta)} = N^{-1} \textstyle\sum_{i=1}^{N} F_{i,\alpha(\beta)} \tag{19b}$$

$$\bar{F}_{\alpha\beta} = N^{-1} \textstyle\sum_{i=1}^{N} F_{i,\alpha\beta} \tag{19c}$$

In practice, u_{it} must be replaced by $\hat{u}_{it} = y_{it} - \hat{\alpha}_{1i} - \hat{\alpha}_{2i} S_{it}(\hat{\gamma}_i, \hat{\tau}_i)$ or $\hat{u}_{it} = y_{it} - \hat{\alpha}_{1i} - \hat{\beta}_{1i} t - \hat{\alpha}_{2i} S_{it}(\hat{\gamma}_i, \hat{\tau}_i)$ or $\hat{u}_{it} = y_{it} - \hat{\alpha}_{1i} - \hat{\beta}_{1i} t - \hat{\alpha}_{2i} S_{it}(\hat{\gamma}_i, \hat{\tau}_i) - \hat{\beta}_{2i} t S_{it}(\hat{\gamma}_i, \hat{\tau}_i)$, where the hat over coefficient denotes nonlinear least squares (NLS) estimator of the corresponding parameters in eq. (2a-2c) and eq.(3). As the NLS estimation does not provide a closed-form solution for the transition parameters, the asymptotic distribution of the test statistics cannot be driven analytically (see, e.g, Leybourne *et al*., 1998). Therefore, the critical values of the test statistics are derived via stochastic simulations. Simulated critical values of the $\bar{F}_{\alpha}$, $\bar{F}_{\alpha(\beta)}$ and $\bar{F}_{\alpha\beta}$ test statistics under the CSD assumption are presented in Table 1. Critical values of these test statistics without CSD are presented in Table 2.

Small sample performance of the proposed test statistics

Finite sample size and power properties of the proposed test statistics are investigated through Monte Carlo simulations. We have also compared the power properties of the proposed test statistics to that of the existing tests. Here we report only small sample performance of the $\bar{F}_{\alpha}$ statistics as the gradually changing mean given in the Model A can be proxied by a linear trend function, especially if the speed of transition is rather small (see, e.g., Omay *et al.,* 2018b). The deterministic trend functions given in Models B and C, on the other hand, are highly nonlinear and cannot be captured by a simple linear model. Therefore, to make comparisons with the other tests "fair", we consider only the $\bar{F}_{\alpha}$ statistics. In fact, our simulations with Models B and C have shown that conventional linear tests have negligible power against trend specifications given in Models B and C, while they preserve some power only against the simpler trend specifications given in Model A. The simulation results reported in Omay *et al.* (2018b), in which the logistic transition functions to model gradual breaks, also conform this finding. Their results also suggest that conventional models have almost no power against highly nonlinear trend specifications given in Models B and C.

The size of the test statistics was assessed using the following data generating process (DGP):

$$y_{it} = y_{it-1} + \varepsilon_{it}$$

with $\varepsilon_{it} \sim iid\ N(0, \sigma_i^2)$, $\varepsilon_{i0} = 0$, and $\sigma_i^2 \sim iid\ U(0.5, 1{,}5)$,

Table 3 reports the empirical size of the proposed test. The results presented in this table suggest that the size of the test is reasonably close to the nominal size. Next, in Table 4S(.), we analyse power of the proposed test in small samples. For comparison, we also examine the power of the panel unit root tests proposed by Im *et al.* (2003) (the IPS test), Ucar and Omay (2009) (the UO test) and Omay *et al.* (2018b) (the OHS test). The IPS test is a generalisation of the conventional ADF test, and thus takes neither structural breaks nor asymmetric adjustment into account. The UO test extends the nonlinear unit root test of Kapetanios *et al.* (2003) to panel data but omits structural breaks. Also notice that the UO test allows for only size asymmetry, but not sign asymmetry in the adjustment process. Finally, the OHS test takes account of the structural break but not asymmetric adjustment.

The power analysis of the mentioned tests is based on the following DGP:

$$y_{it} = \alpha_{1i} + \alpha_{2i}[1 + \exp\{-\gamma_i(t - \tau_i T)\}]^{-1} + u_{it} \quad (21)$$

$$\Delta u_{it} = G_{it}(\theta_{1i}, u_{it-1})\{F_{it}(\theta_{2i}, u_{it-1})\rho_{1i} + (1 - F_{it}(\theta_{2i}, u_{it-1}))\rho_{2i}\}u_{it-1} + \epsilon_{it}$$

$$G_{it}(\theta_{1i}, u_{it-1}) = 1 - \exp\left(-\theta_{1i}\left(u_{it-1}^2\right)\right),$$

$$F_{it}(\theta_{2i}, u_{it-1}) = [1 + \exp(-\theta_{2i}u_{it-1})]^{-1}$$

with $\alpha_{1i} = 1.0$, $\alpha_{2i} = (2.0,10.0)$, $\gamma_i \sim iid\ U(0.1,1.0)$, $\tau_i \sim iid\ U(0.2,0.5)$, $\epsilon_{i0} = 0$, $\epsilon_{it} \sim NID(0,1)$, $\theta_{1i} \sim iid\ U(0.1,1.0), \theta_{2i} = 1.0$, $\rho_{1i} = (-0.05, -0.03)$, and $\rho_{2i} = (-0.05, -1.0)$. These parameter values allow for rich structural break and adjustment dynamics in the data. Specifically, we allow for small ($\alpha_{2i} = 2.0$) and large ($\alpha_{2i} = 10.0$) structural breaks where the speed of break varies from rather slow ($\gamma_i = 0.1$) to moderate ($\gamma_i = 1.0$). Adjustment towards the equilibrium is also modelled to allow for slow to moderate adjustment of deviations from the nonlinear attractor. Note also that we allow for heterogeneity in both break and adjustment processes. The DGP given in eq. (21) with specified parameter values produce data for which conventional unit root tests ignoring either structural break and/or nonlinear adjustment preserve power. It has been documented that such conventional tests have little, if any, power in rather rich settings (see, for example, Omay *et al*., 2018b).

The results reported in Table 4 reveal that the unit root tests ignoring structural break have relatively good power properties only in the case of small breaks when $\alpha_{2i} = 2.0$. Simulation results of Omay *et al.* (2018b) have shown that small and smooth breaks can be approximated by a straight line reasonably well. This explains the relatively good performance of the conventional tests in the case of small breaks. However, when the break is relatively large, these tests perform rather poorly. In particular, the simulation results suggest that the conventional IPS test has no power against large breaks (i.e., when $\alpha_{2i} = 10.0$). Even increasing either the number of the cross-sectional units or the time dimension does not improve the power of the IPS test. In fact, notice that the power of the $\bar{t}_{IPS}$ test is almost 0.0 for all combinations of N and T when $\alpha_{2i} = 10.0$. On the other hand, while the UO test has low power in small panels, power of this test rises with both the time dimension and the number of cross-sectional units. In fact, power of the $\bar{t}_N$ test rises to 0.984 when $N = T = 100$.

The tests that allow for break in the data perform quite satisfactorily in case of both small and large breaks even in small panels. Powers of both the $\bar{t}_{SB}$ test of OHS (Omay *et al*. (2018b) , as well as of the $\bar{F}_{\alpha}$ test proposed in this paper, grow rapidly with the time dimension and the number of cross-sectional units. Note also that the $\bar{F}_{\alpha}$ test outperforms the $\bar{t}_{SB}$ test in the case of small breaks. On the other hand, the $\bar{t}_{SB}$ test outperforms $\bar{F}_{\alpha}$ in case of large breaks in small panels, especially in the case of small time dimension, i.e., when $T \leq 50$. This is an expected result as the $\bar{F}_{\alpha}$ test entails estimation of additional parameters which reduces power of the test in small time dimensions. However, the power of the $\bar{F}_{\alpha}$ test rises quite rapidly with the time dimension and outperforms the $\bar{t}_{SB}$ test for $T \geq 70$. Notice also that the power of both tests rises rapidly with both T and N, as expected. In fact, power of both tests equals 1.000 for $N, T \geq 50$. We have also included explanatory graphics for the power analysis to Technical Annex part B.

Data and unit root tests

Monthly observations on the unemployment rate series of the 19 Eurozone countries covering the 2000:M2-2020:M6 period was retrieved from the Eurostat database.[1] Data was retrieved on 29th of September 2020. We selected the longest possible time span, for which unemployment series of all Euro member countries were available. The data is seasonally adjusted, but not calendar adjusted. In statistical analysis, we use natural logarithms of the series. In addition to testing for stationarity of the unemployment series, we also test for the stochastic convergence to the Euro Area unemployment. Stochastic convergence is usually tested by testing the cointegration of variables or by testing stationarity of a linear combination of the variables. Hence, we also test stationarity of (log of) the relative unemployment rate, defined as $\tilde{u}_{it} = \ln(U_{it}) - \ln(U_{EA19t})$, where U_{it} and U_{EA19t} are the unemployment rate of country i and Euro Area, respectively.

In addition to the $\bar{F}_{\alpha}$, $\bar{F}_{\alpha(\beta)}$ and $\bar{F}_{\alpha\beta}$ test statistics proposed in this paper, we also employ some of the existing panel unit root tests for comparison purposes. In particular, we employ the IPS (Im *et al*.. 2003) test, $\overline{CADF}$ test of Pesaran (2007), nonlinear unit root tests of Ucar and Omay (2009) and Cerrato *et al*. (2007), as well as gradual break test of Omay *et al*. (2018b). The $\overline{CADF}$ test of Pesaran (2007) differs from the IPS (Im *et al*.. 2003) test in that the former adopts the CCE estimator to deal with the CSD problem.

[1] See also U.S. Bureau of Labor Statistics (2020) and European Commission (EC) (2020) for labour data.

Similarly, the Cerrato *et al.* (2007) also uses the CCE estimator, while Ucar and Omay (2009) propose using bootstrap methods to handle the CSD problem. To handle the CSD problem in the IPS (Im *et al..* 2003) test, we use the Sieve bootstrap procedure. Therefore, when applying the gradual break test of Omay *et al.* (2018b) and the newly proposed tests, we also employ both the CCE estimator and bootstrap procedures. Notice that the IPS (Im *et al..* 2003) and the $\overline{CADF}$ test of Pesaran (2007) allows for neither structural breaks nor nonlinearities in the series. However, the Ucar and Omay (2009) and Cerrato *et al*. (2007) tests allow for nonlinear adjustment but not for structural breaks. The test of Omay *et al.* (2018b), on the other hand, allows for breaks while assuming that the adjustment towards equilibrium is linear.

Results

Before testing the stationarity of the desired series, we had estimated the nonlinear trend functions for the unemployment rate and carried out some diagnostic tests. Figure 1 presents the estimated trend function along with unemployment series. Estimated coefficients of the nonlinear trend specification given in Model C are reported in Table 5. As can be seen from the table, estimated coefficients of the trend function are highly statistically significant, suggesting that the deterministic component of the series is inherently nonlinear.

Then we applied linearity tests of Terasvirta (1994) to see whether the nonlinear specification is valid for the data under consideration The test results reported in Table 6 suggest that deviations from the estimated trend function are, in fact, nonlinear. Notice that the null of linearity is rejected at 1% significance level for all countries. We also estimated a nonlinear AESTAR model for the adjustment of deviations from the trend function. Estimated coefficients are reported in Table 7 and estimated transition functions are visualized in Figure 2. Estimated coefficients of the AESTAR models are highly significant and this suggests that all the series can reasonably be modelled with the AESTAR nonlinearity. Finally, to test whether there is cross-section dependence (CSD) in our data, we calculated CD, CD_{LM1}, and CD_{LM2} statistics proposed by Pesaran (2004). The calculated test statistics are $CD = 35.441$ with p-value $p = 0.000$, $CD_{LM1} = 1820.301$ with p-value $p = 0.000$, and $CD_{LM2} = 89.184$ with p-value $p = 0.000$. These results suggest the presence of CSD in our data, and hence CSD-robust test statics were applied throughout this paper.

Although we find that the series are subject to both structural breaks and nonlinearities, we also applied conventional linear tests for comparison purposes. Let us first consider the stationarity of the unemployment rates. The results of the various unit root tests are reported in Table 8. As it can be seen from the table, the test procedures ignoring structural breaks fail to reject the null hypothesis of unit root irrespective of the method chosen to deal with the CSD problem. Both the linear (IPS (Im *et al.*. 2003) and $\overline{CADF}$) and nonlinear (Ucar & Omay, 2009; Cerrato *et al.*, 2007) tests suggest that the panel of unemployment rates of the Eurozone countries are non-stationary. On the other hand, if one allows for a gradual break in the series, the null hypothesis can be rejected against the alternative of linear and nonlinear stationary processes.

Conventional panel unit root test procedures do not provide an explicit guidance of which cross-section entities are stationary. Therefore, we employ the sequential panel selection method (SPSM) proposed by Chortareas and Kapetanios (2009), which enables us to identify stationary cross-section units sequentially. The SPSM procedure starts with testing the null hypothesis of all individual series containing a unit root. If the null is not rejected, the hypothesis is accepted, and the procedure stopped. If the null hypothesis is rejected, one removes the individual series with the lowest p-value from the panel and repeats the panel test with the remaining series. This procedure is continued until either the null is not rejected, or all series are removed from the panel (see also Omay *et al.*, 2018b). As the null of unit root is more convincingly rejected in the case of the Model B for the CCE-based tests, we report only results for this model. The results are reported in Table 10. Both the $\bar{t}_{SB,\alpha(\beta)}$ and $\bar{F}_{\alpha(\beta)}$ rejected the null hypothesis in five out of 19 countries. Notice also that the rejection of the null hypothesis of unit root in the panel is due to presence of a few big (in absolute value) test statistics. After removal of the series with biggest test statistics, the panel tests do not reject the null hypothesis. In fact, after removal of these five series, we were not able to reject the null hypothesis, suggesting that majority of the series contain a unit root.

Now, we turn to the examination of stationarity of unemployment rates relative to the Eurozone unemployment. The results for the unit root tests in the panel of the (log) difference between member countries' unemployment rate and EU19 unemployment rate are provided in Table 9. Again, notice that the unit root tests that do not take either structural breaks or nonlinearities into account fail to detect stationarity in the series. On the other hand, once we allow for a gradual break, we were able to reject the null hypothesis of unit root. All the CCE-based tests, including the newly proposed $\bar{F}_{\alpha}$, $\bar{F}_{\alpha(\beta)}$ and $\bar{F}_{\alpha\beta}$ tests statistics as well as all the corresponding tests of Omay *et al*. (2018b) (i.e., the $\bar{t}_{\alpha}$, $\bar{t}_{\alpha(\beta)}$ and $\bar{t}_{\alpha\beta}$ tests) reject the null of unit root for the relative unemployment series of the Eurozone countries, thus providing some evidence in favour of the convergence of the unemployment rates in the Euro area. However, results of the SPSM procedure that are reported in Table 11, suggest that the newly proposed $\bar{F}_{\alpha\beta}$ test statistics rejects the null hypothesis of unit root in more cases when compared to the $\bar{t}_{\alpha\beta}$ statistics. In particular, the bootstrap based $\bar{F}_{\alpha\beta}$ test has rejected the null hypothesis in 12 out of 19 cases, whereas the bootstrap based $\bar{t}_{\alpha\beta}$ test has rejected it in eight cases. CCE based tests, on the other hand, suggest stationarity in nine cases.

Discussion

This paper develops a new panel unit root test that allows simultaneously for structural breaks and nonlinear adjustment towards the equilibrium in the unemployment series. Monte Carlo simulation results reveal that conventional unit root tests have almost no power in case of big structural breaks, suggesting that researchers ignoring possible breaks may lead to misleading results. Also, we find that the test procedures that explicitly model structural breaks preserve relatively good power in small samples against nonlinear adjustment as well. On the other hand, tests modelling only nonlinearities but not structural breaks, have little power in relatively small samples whereas achieve reasonable power in large sample (typically for $N, T \geq 100$).

We apply the newly proposed test to examine the stationarity of individual and relative unemployment rates. We also apply conventional unit root tests for comparison purposes. Before applying the unit root tests, we estimate the nonlinear trend function for the series under consideration and carry out some diagnostic tests. The results suggest that the unemployment rates of the sample countries are highly nonlinear. Also, we find strong evidence of cross-country correlation in the series. Overall, our finding implies that the dynamics of the unemployment series are inherently highly nonlinear.

Our main findings can be summarized as follows. First, the panel unit root tests that do not account for the presence of structural break fail to detect stationarity in the data, irrespective of whether nonlinearity is accounted for or not. In fact, the bootstrap-based and CCE-based tests reject the unit root null neither in the panel of unemployment nor in the panel of relative unemployment series. However, in line with León-Ledesma and

McAdam (2004), Lee and Chang (2008), Furuoka (2017), Jiang *et al.* (2019), Krištić *et al.* (2019) and Omay *et al.* (2021), the null is rejected once we allow for gradual breaks in the series. Second, we find that allowing for the possibility of nonlinear adjustment towards the equilibrium results in more frequent rejection of the null hypothesis. Thus, our results corroborate the findings of, among others, Caner and Hansen (2001), Franchi and Ordonez (2008), Lee (2010), and Meng *et al*. (2017). Third, our results provide weak evidence for stationarity of the unemployment series while stronger evidence in favour of the stationarity of the relative unemployment rates, thus supporting the stochastic convergence hypothesis among the unemployment series. Similarly, Kónya (2020), Krištić *et al*. (2019), and Carrera and Rodriguez (2009) using different sample periods and country groups have also verified the existence of stochastic convergence for most of the EU countries.

Conclusions

In this paper, we have examined the stationarity and stochastic convergence of unemployment series in the Eurozone. Taking account of the fact that most economic variables are characterized by both structural breaks and nonlinear adjustment, we have proposed a new procedure to test for stationarity in the panel data. Structural breaks in the series are modelled by the logistic transition function which allow for a gradual break in the mean and trend of the series. Adjustment towards the gradually changing trend is modelled using the asymmetric exponential function which allows the speed of the convergence towards the attractor to vary with both the size and sign of deviations from the attractor. Thus, the proposed test procedure allows for rather rich dynamics observed in most economic series. Using small scale stochastic simulations, we show that the proposed tests have desirable small sample performance.

Our results have clear and peculiar policy implications. First, our results provide further evidence in favour of the hysteresis hypothesis in most of the Euro-member countries, thus providing an empirical justification for proactive economic policy to fight unemployment, especially during hard economic times. Policy inaction in face of recessions can cause irreversible output and employment losses in the presence of hysteresis. However, the highly nonlinear dynamics of the unemployment series implies that these dynamics of unemployment and mechanisms through which economic policy actions affect unemployment must thoroughly be analysed before taking policy actions. For example, if the adjustment of unemployment is dependent on the size of deviations, timid policy actions may have no effect at all. While the possibility of structural breaks in the future questions effectiveness of the monetary and fiscal policies to control the unemployment rate effectively, it also provides a support for non-conventional policies such as banning or restricting layoffs during unprecedented and temporary real shocks like the recent pandemic. With hysteresis, there is no room for policy mistakes, since any mistake in face of a shock as large as the one that we are now facing will impose irreversible costs to the societies. If we cannot manage this pandemic with appropriate policy stimuli, then unfortunately it can leave permanent scars on unemployment. Second, the fact that we were able to find some support for stochastic convergence among the Eurozone countries *only after* allowing for structural breaks, leaves little room for coordinated economic policy. In fact, the economic policy favourable for one country will be unfavourable for the others if there are sharp breaks in the common trend of economic variables. Possible nonlinearities in the adjustment of individual series towards this common trend further weaken justifications for the common policies.

Our study has some limitations. First, our study does not address the possible reasons of structural breaks of unemployment series. Although it is known that big real shocks may cause big shifts in the level of unemployment, it would be interesting to analyse the extents of such shifts for individual countries. Also, it would be compelling to analyse the nature and factors causing the co-movement of deviations from the long-run trend for Eurozone countries. Finally, the world economy in general and eurozone countries in particular, have suffered several real shocks during the last two decades. It would be interesting to see how these shocks affected unemployment dynamics and co-movement in the sample countries. We believe that the procedures developed in this paper will provide useful tools for addressing these and other issues in the future.

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Annex

Table 1. Critical values of CSD-robust $\bar{F}_{\alpha}$, $\bar{F}_{\alpha(\beta)}$ and $\bar{F}_{\alpha\beta}$

	N (number of cross-section entities)						
T Time dimension	5	10	15	20	25	50	100
Model A: Break in the mean without trend: $\boldsymbol{\phi_{it} = \alpha_{1i} + \alpha_{2i}S_{it}(\gamma_i, \tau_i)}$							
1% critical values of the $\bar{F}_{\alpha}$ statistics							
30	9.449	7.855	7.534	7.114	6.972	6.338	5.979
50	8.892	7.437	6.976	6.745	6.607	6.153	5.737
70	8.635	7.281	6.905	6.550	6.334	6.035	5.628
100	8.497	7.039	6.616	6.519	6.227	5.878	5.574
5% critical values of the $\bar{F}_{\alpha}$ statistics							
30	7.776	7.010	6.709	6.474	6.428	5.967	5.784
50	7.518	6.644	6.358	6.149	6.119	5.809	5.536
70	7.355	6.513	6.296	6.088	5.964	5.658	5.479
100	7.178	6.397	6.185	6.016	5.890	5.585	5.412
10% critical values of the $\bar{F}_{\alpha}$ statistics							
30	7.068	6.582	6.269	6.147	6.125	5.799	5.668
50	6.850	6.243	6.134	5.876	5.835	5.628	5.422
70	6.666	6.156	6.046	5.830	5.764	5.511	5.382
100	6.503	6.061	5.896	5.706	5.673	5.436	5.342
Model B: Break in the mean with trend: $\boldsymbol{\phi_{it} = \alpha_{1i} + \beta_{1i}t + \alpha_{2i}S_{it}(\gamma_i, \tau_i)}$							
1% critical values of the $\bar{F}_{\alpha(\beta)}$ statistics							
30	12.497	10.580	10.044	9.564	9.205	8.710	8.288
50	11.431	9.624	8.802	8.729	8.342	7.897	7.542
70	10.441	8.772	8.614	8.483	8.104	7.682	7.447
100	10.337	8.548	8.404	8.254	7.983	7.546	7.214
5% critical values of the $\bar{F}_{\alpha(\beta)}$ statistics							
30	10.505	9.460	9.087	8.709	8.555	8.226	7.939
50	9.723	8.629	8.194	8.016	7.825	7.574	7.328
70	9.190	8.271	7.956	7.799	7.639	7.385	7.186
100	9.053	8.153	7.805	7.746	7.519	7.225	7.016
10% critical values of the $\bar{F}_{\alpha(\beta)}$ statistics							
30	9.611	8.900	8.582	8.300	8.287	7.963	7.718
50	8.899	8.155	7.848	7.698	7.590	7.387	7.183
70	8.499	7.842	7.665	7.543	7.414	7.161	7.039
100	8.414	7.660	7.507	7.443	7.283	7.053	6.889
Model C: Break in both the mean and trend: $\boldsymbol{\phi_{it} = \alpha_{1i} + \beta_{1i}t + \alpha_{2i}S_{it}(\gamma_i, \tau_i) + \beta_{2i}tS_{it}(\gamma_i, \tau_i)}$							
1% critical values of the $\bar{F}_{\alpha\beta}$ statistics							
30	13.051	12.870	12.232	11.410	11.314	10.442	10.096
50	12.265	11.387	10.602	10.488	10.120	9.415	9.088
70	11.947	10.665	9.988	9.845	9.629	9.015	8.753
100	11.665	10.290	9.810	9.459	9.261	8.809	8.528

Table 1. Continued

5% critical values of the $\bar{F}_{\alpha\beta}$ statistics							
30	11.939	11.552	11.095	10.683	10.493	10.000	9.772
50	11.410	10.309	9.905	9.665	9.457	9.002	8.811
70	10.629	9.772	9.383	9.185	9.049	8.642	8.459
100	10.375	9.424	9.029	8.876	8.717	8.470	8.307
10% critical values of the $\bar{F}_{\alpha\beta}$ statistics							
30	11.475	10.845	10.545	10.217	10.104	9.717	9.470
50	10.394	9.737	9.443	9.253	9.118	8.813	8.654
70	9.908	9.343	8.987	8.886	8.738	8.428	8.306
100	9.716	9.001	8.777	8.570	8.455	8.287	8.169

Table 2. Critical values of no-CSD $\bar{F}_{\alpha}$, $\bar{F}_{\alpha(\beta)}$ and $\bar{F}_{\alpha\beta}$

	N (number of cross-section entities)						
T Time dimension	5	10	15	20	25	50	100
Model A: Break in the mean without trend: $\boldsymbol{\phi_{it} = \alpha_{1i} + \alpha_{2i} S_{it}(\gamma_i, \tau_i)}$							
1% critical values of the $\bar{F}_{\alpha}$ statistics							
30	9.027	7.734	7.119	6.741	6.520	6.136	5.749
50	7.959	6.989	6.744	6.243	6.030	5.678	5.397
70	7.407	6.868	6.314	6.035	5.955	5.511	5.276
100	7.315	6.456	6.205	5.891	5.841	5.354	5.138
5% critical values of the $\bar{F}_{\alpha}$ statistics							
30	7.440	6.837	6.416	6.181	6.088	5.760	5.523
50	6.808	6.177	5.953	5.782	5.684	5.367	5.194
70	6.378	6.096	5.796	5.635	5.479	5.240	5.087
100	6.419	5.896	5.680	5.504	5.387	5.131	5.001
10% critical values of the $\bar{F}_{\alpha}$ statistics							
30	6.778	6.370	6.067	5.899	5.797	5.599	5.395
50	6.344	5.837	5.639	5.534	5.452	5.244	5.089
70	5.884	5.707	5.512	5.389	5.294	5.109	4.995
100	5.986	5.599	5.390	5.265	5.201	5.007	4.898
Model B: Break in the mean with trend: $\boldsymbol{\phi_{it} = \alpha_{1i} + \beta_{1i} t + \alpha_{2i} S_{it}(\gamma_i, \tau_i)}$							
1% critical values of the $\bar{F}_{\alpha(\beta)}$ statistics							
30	11.705	10.252	9.417	9.126	8.943	8.345	7.903
50	10.077	8.997	8.299	8.133	8.119	7.453	7.078
70	9.614	8.546	8.099	7.857	7.541	7.172	6.879
100	9.183	8.299	7.727	7.468	7.406	6.989	6.686
5% critical values of the $\bar{F}_{\alpha(\beta)}$ statistics							
30	9.895	8.983	8.624	8.462	8.318	7.840	7.665
50	8.910	8.134	7.728	7.620	7.499	7.144	6.918
70	8.401	7.824	7.444	7.359	7.182	6.857	6.654
100	8.165	7.547	7.188	7.034	6.996	6.695	6.485
10% critical values of the $\bar{F}_{\alpha(\beta)}$ statistics							
30	9.129	8.482	8.223	8.067	7.962	7.641	7.502
50	8.276	7.750	7.438	7.325	7.212	6.971	6.811
70	7.866	7.450	7.142	7.092	6.950	6.710	6.571
100	7.640	7.200	6.934	6.820	6.765	6.539	6.366

Table 2. Continued

	N (number of cross-section entities)						
T Time dimension	5	10	15	20	25	50	100
Model C: Break in the both the mean and trend: $\boldsymbol{\phi_{it} = \alpha_{1i} + \beta_{1i}t + \alpha_{2i}S_{it}(\gamma_i, \tau_i) + \beta_{2i}tS_{it}(\gamma_i, \tau_i)}$							
1% critical values of the $\bar{F}_{\alpha\beta}$ statistics							
30	14.054	12.641	11.649	11.288	11.029	10.396	9.959
50	12.173	10.625	10.139	9.877	9.550	8.980	8.635
70	11.425	10.392	9.420	9.398	9.006	8.622	8.213
100	10.707	9.762	9.132	8.994	8.597	8.140	7.939
5% critical values of the $\bar{F}_{\alpha\beta}$ statistics							
30	12.377	11.203	10.758	10.360	10.347	9.841	9.570
50	10.851	9.745	9.353	9.227	8.998	8.662	8.411
70	9.972	9.345	8.911	8.735	8.506	8.165	8.018
100	9.524	8.880	8.656	8.332	8.239	7.815	7.727
5% critical values of the $\bar{F}_{\alpha\beta}$ statistics							
30	11.366	10.599	10.262	9.983	9.895	9.602	9.332
50	10.000	9.306	8.931	8.882	8.718	8.428	8.265
70	9.371	8.862	8.487	8.441	8.265	7.950	7.903
100	8.963	8.490	8.353	8.068	8.029	7.590	7.638

Table 3. Small sample size of the $\bar{F}_{\alpha}$ test statistics

	N (number of cross-section entities)			
T Time dimension	**5**	**25**	**50**	**100**
30	0.045	0.045	0.046	0.045
50	0.047	0.048	0.049	0.048
70	0.052	0.050	0.049	0.051
100	0.051	0.052	0.051	0.050

Table 4. Small sample power analysis of the alternative test statistics

Panel A- Tests ignoring structural break								
Small gradual break $\alpha_{2i} = 2.0$								
	N = 5		**N = 25**		**N=50**		**N=100**	
T	$\bar{t}_{IPS}$	$\bar{t}_N$	$\bar{t}_{IPS}$	$\bar{t}_N$	$\bar{t}_{IPS}$	$\bar{t}_N$	$\bar{t}_{IPS}$	$\bar{t}_N$
30	0.166	0.312	0.534	0.898	0.812	0.994	0.978	0.998
50	0.472	0.726	0.976	1.000	1.000	1.000	1.000	1.000
70	0.736	0.950	1.000	1.000	1.000	1.000	1.000	1.000
100	0.978	0.998	1.000	1.000	1.000	1.000	1.000	1.000
Large gradual break $\alpha_{2i} = 10.0$								
	N = 5		**N = 25**		**N=50**		**N=100**	
T	$\bar{t}_{IPS}$	$\bar{t}_N$	$\bar{t}_{IPS}$	$\bar{t}_N$	$\bar{t}_{IPS}$	$\bar{t}_N$	$\bar{t}_{IPS}$	$\bar{t}_N$
30	0.000	0.006	0.000	0.016	0.000	0.022	0.000	0.067
50	0.000	0.018	0.000	0.040	0.000	0.064	0.000	0.117
70	0.000	0.028	0.000	0.062	0.000	0.111	0.000	0.181
100	0.000	0.132	0.003	0.454	0.000	0.866	0.000	0.984

Table 4. Continued

Panel B- Tests allowing for gradual break								
Small gradual break $\alpha_{2i} = 2.0$								
	N = 5		**N = 25**		**N=50**		**N=100**	
T	$\bar{t}_{SB}$	$\bar{F}_{\alpha}$	$\bar{t}_{SB}$	$\bar{F}_{\alpha}$	$\bar{t}_{SB}$	$\bar{F}_{\alpha}$	$\bar{t}_{SB}$	$\bar{F}_{\alpha}$
30	0.156	0.192	0.446	0.674	0.662	0.918	0.910	1.000
50	0.358	0.530	0.928	0.996	1.000	1.000	1.000	1.000
70	0.598	0.840	1.000	1.000	1.000	1.000	1.000	1.000
100	0.946	0.996	1.000	1.000	1.000	1.000	1.000	1.000
Large gradual break $\alpha_{2i} = 10.0$								
	N = 5		**N = 25**		**N=50**		**N=100**	
T	$\bar{t}_{SB}$	$\bar{F}_{\alpha}$	$\bar{t}_{SB}$	$\bar{F}_{\alpha}$	$\bar{t}_{SB}$	$\bar{F}_{\alpha}$	$\bar{t}_{SB}$	$\bar{F}_{\alpha}$
30	0.130	0.064	0.402	0.140	0.668	0.280	0.932	0.509
50	0.384	0.288	0.974	0.894	1.000	1.000	1.000	1.000
70	0.706	0.738	1.000	1.000	1.000	1.000	1.000	1.000
100	0.946	0.996	1.000	1.000	1.000	1.000	1.000	1.000

Notes: T is the time dimension and N is the number of cross-section entities.
The empirical powers of the tests are evaluated using bootstrap re-sampling with 2000 replications. $\bar{t}_{IPS}$, $\bar{t}_{N}$, and $\bar{t}_{SB}$ denote the IPS (Im *et al.,* 2003), UO (Ucar & Omay, 2009) and OHS (Omay *et al.*, 2018b) tests, respectively, whereas $\bar{F}_{\alpha}$ is the test statistics given in eq. (19a). Note that the $\bar{t}_{IPS}$ statistics take account of neither structural breaks nor asymmetric adjustment. The $\bar{t}_{N}$ test allows for nonlinear adjustment but fails to consider structural breaks. The $\bar{t}_{SB}$ statistics, on the other hand, model gradual breaks but ignore nonlinearities in the adjustment process.

Table 5. Estimated coefficients of the nonlinear trend function

	$\boldsymbol{\alpha_1}$	$\boldsymbol{\alpha_2}$	$\boldsymbol{\beta_1}$	$\boldsymbol{\beta_2}$	$\boldsymbol{\gamma}$	$\boldsymbol{\tau}$
Austria	-0.738*	0.744*	0.009**	-0.008*	5.967**	0.490*
Belgium	-0.203**	2.647*	0.005*	-0.025**	0.158***	0.851*
Cyprus	-3.673*	41.263*	0.004*	-0.167*	0.114**	0.616*
Estonia	6.153*	9.665**	-0.113*	0.023*	0.414*	0.436**
Finland	1.892*	-0.836*	-0.033*	0.027*	0.638*	0.449*
France	-0.145*	13.062**	-0.009*	-0.050**	0.050**	0.645*
Germany	0.273**	3.511*	0.064**	-0.095***	0.146*	0.321**
Greece	-4.226*	44.470*	-0.033*	-0.132**	0.108**	0.572*
Ireland	-3.526*	27.981*	-0.002*	-0.118**	0.133*	0.462**
Italy	0.775***	16.908**	-0.045***	-0.029*	0.056*	0.587*
Latvia	4.181*	15.478***	-0.096**	-0.010**	0.444**	0.439**
Lithuania	8.227*	9.799*	-0.162**	0.062**	0.279*	0.438*
Luxembourg	-2.713*	1.588*	0.031*	-0.020**	0.664**	0.459*
Malta	1.462**	3.150*	-0.011**	-0.019*	0.741*	0.454*
Netherlands	-0.960*	15.547*	0.007*	-0.076*	0.203**	0.643*
Portugal	-5.005*	32.901*	0.053***	-0.187**	0.255***	0.596*
Slovakia	8.461*	6.205*	-0.105*	0.013**	0.245*	0.481**
Slovenia	0.236**	19.054*	-0.020*	-0.072*	0.082**	0.577*
Spain	-2.997*	40.977*	-0.066**	-0.104*	0.069***	0.501**

Notes: *, ** and *** denote significance at 1%, 5% and 10% significance levels, respectively.

Table 6. Linearity tests results

	Linearity test	**Result**
Austria	12.013*	Nonlinear
Belgium	52.982*	Nonlinear
Cyprus	16.150*	Nonlinear
Estonia	9.663*	Nonlinear
Finland	30.261*	Nonlinear
France	6.865*	Nonlinear
Germany	20.843*	Nonlinear
Greece	15.698*	Nonlinear
Ireland	23.320*	Nonlinear
Italy	7.985*	Nonlinear
Latvia	46.554*	Nonlinear
Lithuania	48.763*	Nonlinear
Luxembourg	14.424*	Nonlinear
Malta	14.305*	Nonlinear
Netherlands	8.715*	Nonlinear
Portugal	11.968*	Nonlinear
Slovakia	29.373*	Nonlinear
Slovenia	38.811*	Nonlinear
Spain	62.937*	Nonlinear

Notes: *, ** and *** denote significance at 1%, 5% and 10% significance levels, respectively.

Table 7. Estimated parameters of the asymmetric AESTAR model

	$\boldsymbol{\rho_1}$	$\boldsymbol{\rho_2}$	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$
Austria	1.927*	1.539*	0.898*	0.950*
Belgium	1.450*	1.489*	0.974*	0.873*
Cyprus	-0.089***	2.749*	1.512*	0.834*
Estonia	0.314*	1.249*	1.380*	0.796*
Finland	1.357*	2.075*	0.981*	0.919*
France	1.310***	1.613*	1.215*	0.681*
Germany	-0.021	7.270*	1.402*	1.560*
Greece	-0.064	1.865**	3.657*	0.594*
Ireland	0.093*	3.067*	0.868*	1.500*
Italy	-0.049	1.821**	2.836*	1.059*
Latvia	0.175*	1.411*	0.735*	0.863*
Lithuania	0.159*	1.553*	1.064*	1.267*
Luxembourg	0.351*	3.998*	1.158*	0.965*
Malta	0.229**	4.427*	1.351*	0.818*
Netherlands	1.373*	1.760*	0.988*	0.792*
Portugal	0.017	2.111*	1.434*	0.847*
Slovakia	0.414*	1.309*	1.175*	0.919*
Slovenia	0.061	2.423*	1.293*	1.047*
Spain	0.067**	1.562*	1.273*	0.943*

Notes: *, ** and *** denote significance at 1%, 5% and 10% significance levels, respectively.

Table 8. Testing Stationary of the Euro Unemployment Rates

Panel A- Tests ignoring structural break						
	IPS (Im *et al.*, 2003)		Ucar and Omay (2009)			
	$\bar{t}_{IPS,c}$	$\bar{t}_{IPS,c\&t}$	$\bar{t}_{N,c}$	$\bar{t}_{N,c\&t}$		
Bootstrap-based tests	-1.067	-1.697	-1.611	-1.433		
	(0.980)	(0.992)	(0.613)	(0.995)		
	Pesaran (2007)		Cerrato *et al.* (2009)			
	$\overline{\overline{CADF}}_c$	$\overline{\overline{CADF}}_{c\&t}$	$\bar{t}_{N,c}$	$\bar{t}_{N,c\&t}$		
CCE-based tests	-1.925	-1.741	-0.904	-0.139		
Panel B- Tests allowing for gradual break						
	Omay *et al.* (2018b)			Newly proposed tests		
	$\bar{t}_{SB,\alpha}$	$\bar{t}_{SB,\alpha(\beta)}$	$\bar{t}_{SB,\alpha(\beta)}$	$\bar{F}_{\alpha}$	$\bar{F}_{\alpha(\beta)}$	$\bar{F}_{\alpha\beta}$
Bootstrap-based tests	-2.830	-3.430	-3.264	4.213	7.959*	8.731**
	(0.487)	(0.330)	(0.970)	(0.660)	(0.098)	(0.021)
CCE-based tests	-3.026*	-4.105***	-4.206***	4.587	10.494***	9.287***

Notes: p-values of the bootstrap-based tests are calculated with 2000 replications and shown below the test statistics. $\bar{t}_{IPS,*}$, $\bar{t}_{N,*}$, $\overline{CADF}_{*}$ denote the IPS (Im *et al.,* 2003), UO (Ucar & Omay, 2009), Pesaran (2007) tests, respectively. The subscripts c and $c\&t$ denote constant and constant and trend. $\bar{t}_{SB,\alpha}$, $\bar{t}_{SB,\alpha(\beta)}$, and $\bar{t}_{SB,\alpha(\beta)}$ denote unit root test statistics proposed by Omay *et al.* (2018b) tests with break in the mean without trend, break in the mean and linear trend, and break both in the mean and trend, respectively. $\bar{F}_{\alpha}$, $\bar{F}_{\alpha(\beta)}$ and $\bar{F}_{\alpha\beta}$ test statistics are defined in eq. (19). *, ** and *** denote rejection the null hypothesis of unit root at 10%, 5% and 1% significance levels, respectively.

Table 9. Testing Stationary of Relative Euro Unemployment Rates

Panel A- Tests ignoring structural break						
	IPS (Im *et al.*, 2003)		Ucar and Omay (2009)			
	$\bar{t}_{IPS,c}$	$\bar{t}_{IPS,c\&t}$	$\bar{t}_{N,c}$	$\bar{t}_{N,c\&t}$		
Bootstrap-based tests	-1.699	-1.787	-2.216	-1.997		
	(0.253)	(0.932)	(0.125)	(0.845)		
	Pesaran (2007)		Cerrato *et al.* (2009)			
	$\overline{\overline{CADF}}_c$	$\overline{\overline{CADF}}_{c\&t}$	$\bar{t}_{N,c}$	$\bar{t}_{N,c\&t}$		
CCE-based tests	-1.682	-2.275	-0.762	-0.934		
Panel B- Tests allowing for gradual break						
	Omay *et al.* (2018b)			Newly proposed tests		
	$\bar{t}_{SB,\alpha}$	$\bar{t}_{SB,\alpha(\beta)}$	$\bar{t}_{SB,\alpha(\beta)}$	$\bar{F}_{\alpha}$	$\bar{F}_{\alpha(\beta)}$	$\bar{F}_{\alpha\beta}$
Bootstrap-based tests	-2.559	-3.611*	-4.486**	6.400	10.393**	15.617**
	(0.858)	(0.089)	(0.019)	(0.112)	(0.029)	(0.021)
CCE-based tests	-3.000*	-4.079***	-4.422***	8.161***	11.276***	15.138***

Notes: p-values of the bootstrap-based tests are calculated with 2000 replications and shown below the test statistics. $\bar{t}_{IPS,*}$, $\bar{t}_{N,*}$, $\overline{CADF}_{*}$ denote the IPS (Im *et al.,* 2003), UO (2009), Pesaran (2007) tests, respectively. The subscripts c and $c\&t$ denote constant and constant and trend. $\bar{t}_{SB,\alpha}$, $\bar{t}_{SB,\alpha(\beta)}$, and $\bar{t}_{SB,\alpha(\beta)}$ denote unit root test statistics proposed by Omay *et al.* (2018b) tests with break in the mean without trend, break in the mean and linear trend, and break both in the mean and trend, respectively. $\bar{F}_{\alpha}$, $\bar{F}_{\alpha(\beta)}$ and $\bar{F}_{\alpha\beta}$ test statistics are defined in eq. (19). *, ** and *** denote rejection the null hypothesis of unit root at 10%, 5% and 1% significance levels, respectively.

Table 10. Testing stationarity of Euro unemployment series using the SPSM procedure

Panel A. Unit Root Tests under Structural Break but Linear Adjustment		
CCE Test		
$\bar{t}_{SB,\alpha(\beta)}$	$\bar{t}_{SB,\alpha(\beta)}$	$\bar{t}_{SB,\alpha(\beta)}$
-4.105***	-7.661	Belgium
-3.908***	-5.912	Estonia
-3.790***	-5.814	Slovenia
-3.663***	-5.391	Malta
-3.548*	-5.010	Ireland
-3.444	-4.965	Luxembourg
-3.327	-4.848	Latvia
-3.200	-4.724	Lithuania
-3.062	-4.383	Austria
-2.929	-4.263	Greece
-2.781	-3.836	Italy
-2.650	-3.533	Portugal
-2.523	-3.415	Spain
-2.375	-2.981	Slovakia
-2.254	-2.790	Finland
-2.120	-2.786	France
-1.898	-2.390	Netherlands
-1.652	-1.814	Cyprus
-1.490	-1.491	Germany

Panel B. Unit Root Tests under Structural Break And Nonlinear Adjustment					
Bootstrap Test			CCE Test		
$\bar{F}_{\alpha(\beta)}$	Maximum test statistic	Country	$\bar{F}_{\alpha(\beta)}$	Maximum test statistic	Country
7.959***	28.728	Greece	10.494***	32.806	Finland
6.806***	18.048	Spain	9.255***	25.855	Greece
6.144***	12.264	Netherlands	8.278***	20.987	Latvia
5.762**	11.435	Slovenia	7.484*	20.562	Belgium
5.384*	10.123	Italy	6.613	19.951	France
5.045	9.715	Belgium	5.660	18.379	Austria
4.686	8.668	Ireland	4.682	9.087	Estonia
4.354	7.121	Malta	4.315	8.700	Italy
4.103	5.675	Estonia	3.916	7.448	Malta
3.946	5.467	Luxembourg	3.563	7.324	Ireland
3.777	5.176	Slovakia	3.145	7.110	Lithuania
3.602	4.937	Latvia	2.649	6.523	Slovenia
3.411	4.627	Lithuania	2.096	5.729	Slovakia
3.209	4.414	Austria	1.491	3.687	Luxembourg
2.967	3.885	Cyprus	1.051	1.635	Portugal
2.738	3.709	France	0.905	1.362	Spain
2.415	3.130	Portugal	0.753	1.042	Netherlands
2.058	3.032	Finland	0.608	0.678	Cyprus
1.084	1.085	Germany	0.539	0.539	Germany

Notes: *,**,*** denotes rejection of the null hypothesis of unit root at 10%, 5% and 1% significance levels, respectively.

Table 11. Testing stationarity of relative Euro unemployment series using the SPSM procedure

Panel A. Unit Root Tests under Structural Break but Linear Adjustment					
Bootstrap Test			**CCE Test**		
$\bar{t}_{SB,\alpha(\beta)}$	Minimum test statistic	Country	$\bar{t}_{SB,\alpha(\beta)}$	Minimum test statistic	Country
-4.486***	-6.822	Belgium	-4.422***	-7.290	Belgium
-4.356***	-6.375	Italy	-4.263***	-6.295	Estonia
-4.237***	-6.081	Netherlands	-4.143***	-5.613	Austria
-4.122***	-5.037	Cyprus	-4.052***	-5.556	Slovenia
-4.061***	-4.970	Ireland	-3.951***	-5.250	Spain
-3.995**	-4.948	Slovenia	-3.859***	-5.165	Latvia
-3.922**	-4.898	Lithuania	-3.758***	-5.156	Malta
-3.841*	-4.68	Malta	-3.642**	-5.069	Ireland
-3.764	-4.680	Greece	-3.512*	-5.046	Italy
-3.672	-4.679	Austria	-3.358	-4.655	Lithuania
-3.561	-4.294	Spain	-3.214	-4.078	Luxembourg
-3.469	-4.238	Luxembourg	-3.106	-3.908	Slovakia
-3.359	-4.201	Estonia	-2.992	-3.827	Cyprus
-3.219	-3.993	Slovakia	-2.853	-3.599	Portugal
-3.064	-3.975	Portugal	-2.703	-3.533	Germany
-2.836	-3.544	Germany	-2.496	-2.786	Netherlands
-2.600	-3.399	France	-2.399	-2.627	France
-2.201	-3.149	Latvia	-2.285	-2.541	Greece
-1.253	-1.253	Finland	-2.029	-2.029	Finland

Panel B. Unit Root Tests under Structural Break And Nonlinear Adjustment					
Bootstrap Test			**CCE Test**		
$\bar{F}_{\alpha(\beta)}$	Maximum test statistic	Country	$\bar{F}_{\alpha(\beta)}$	Maximum test statistic	Country
15.617***	45.656	Portugal	15.138***	43.584	Portugal
13.948***	39.977	Italy	13.558***	32.943	France
12.417***	20.555	Netherlands	12.417***	29.314	Ireland
11.908***	19.192	France	11.361***	24.773	Estonia
11.422***	18.216	Greece	10.467***	19.243	Austria
10.937***	18.210	Cyprus	9.840***	18.079	Italy
10.378***	17.049	Ireland	9.207***	17.418	Slovakia
9.822***	15.989	Slovakia	8.522***	17.042	Belgium
9.261***	15.765	Belgium	7.748***	16.370	Cyprus
8.611**	14.994	Spain	6.886	15.529	Slovenia
7.902*	13.043	Slovenia	5.925	13.959	Latvia
7.259*	11.945	Luxembourg	4.921	8.114	Spain
6.589	9.989	Malta	4.465	7.707	Malta
6.023	9.248	Estonia	3.925	6.454	Greece
5.378	7.796	Lithuania	3.419	5.170	Lithuania
4.773	7.389	Austria	2.981	3.444	Germany
3.901	4.170	Germany	2.827	3.438	Luxembourg
3.767	3.921	Latvia	2.521	3.386	Netherlands
3.613	3.613	Finland	1.656	1.656	Finland

Notes: *,**,*** denotes rejection of the null hypothesis of unit root at 10%, 5% and 1% significance levels, respectively. As the null of unit root in relative unemployment rate is more convincingly rejected in the case of the Model C, we report results only for this model.

Figure 1. Estimated trend functions (blue line) and unemployment rates (black line)

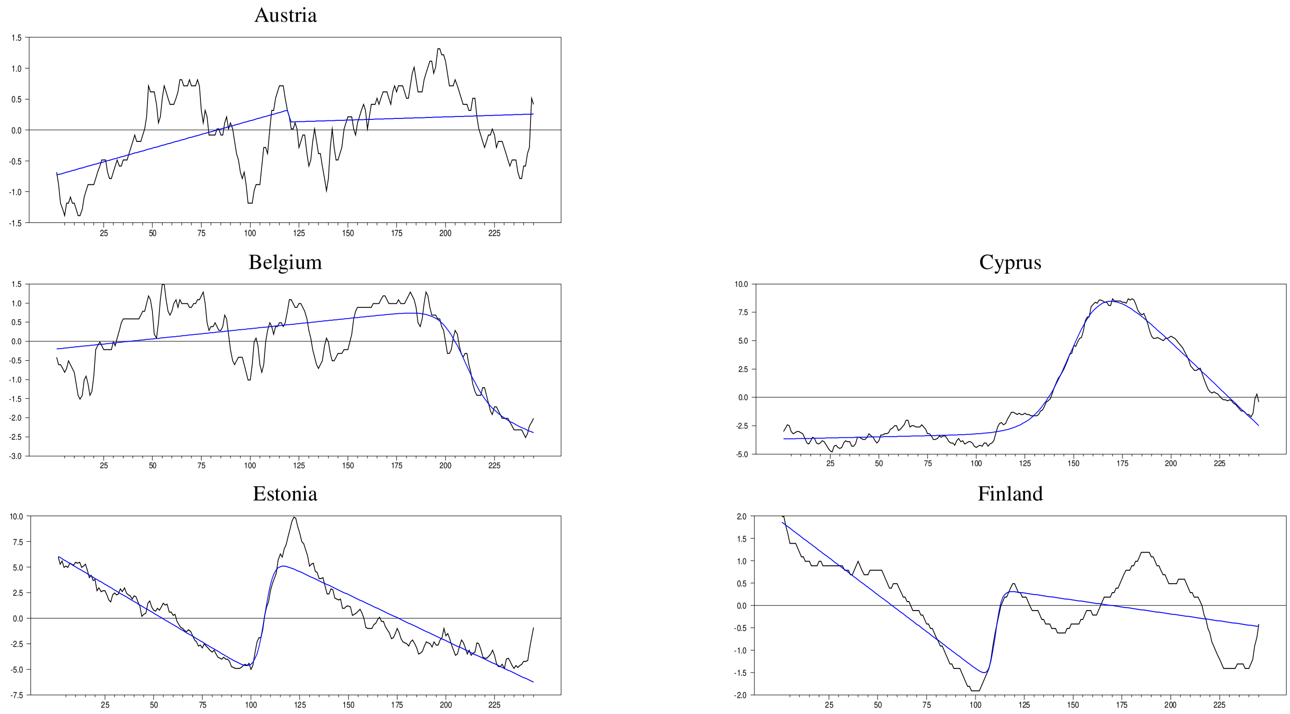


Figure 1. Continued

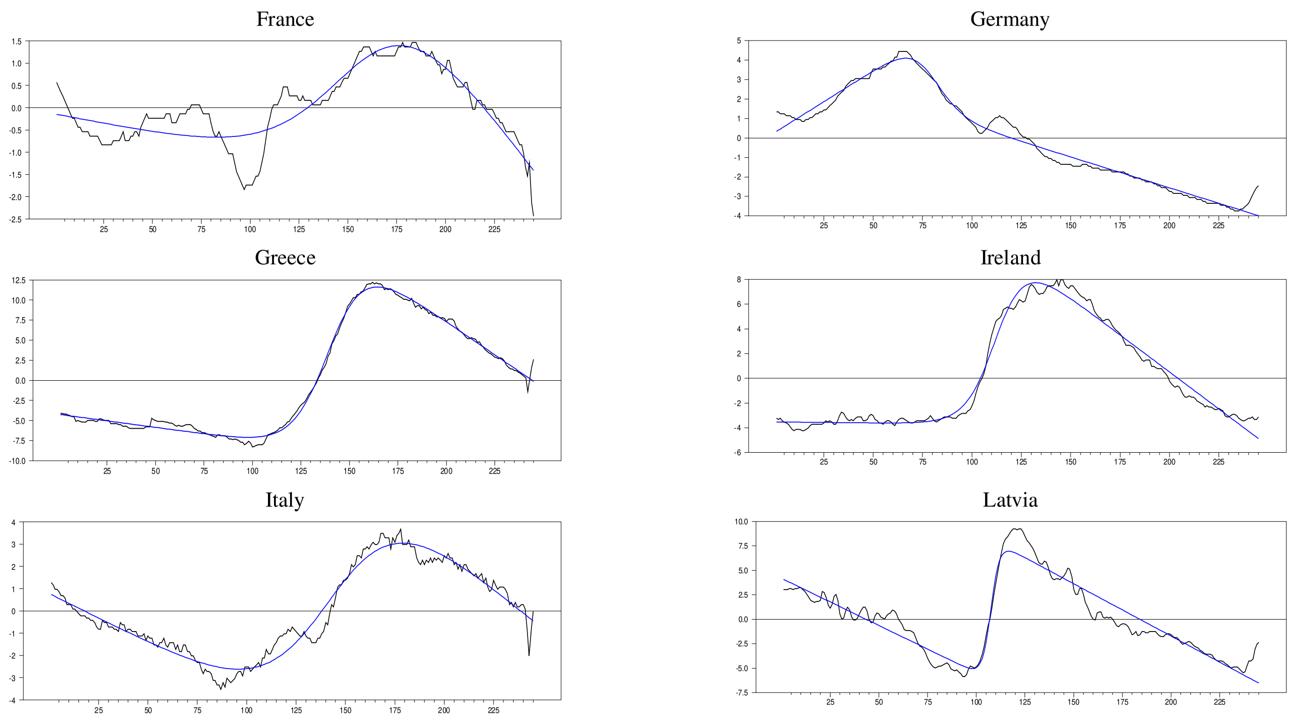


Figure 1. Continued

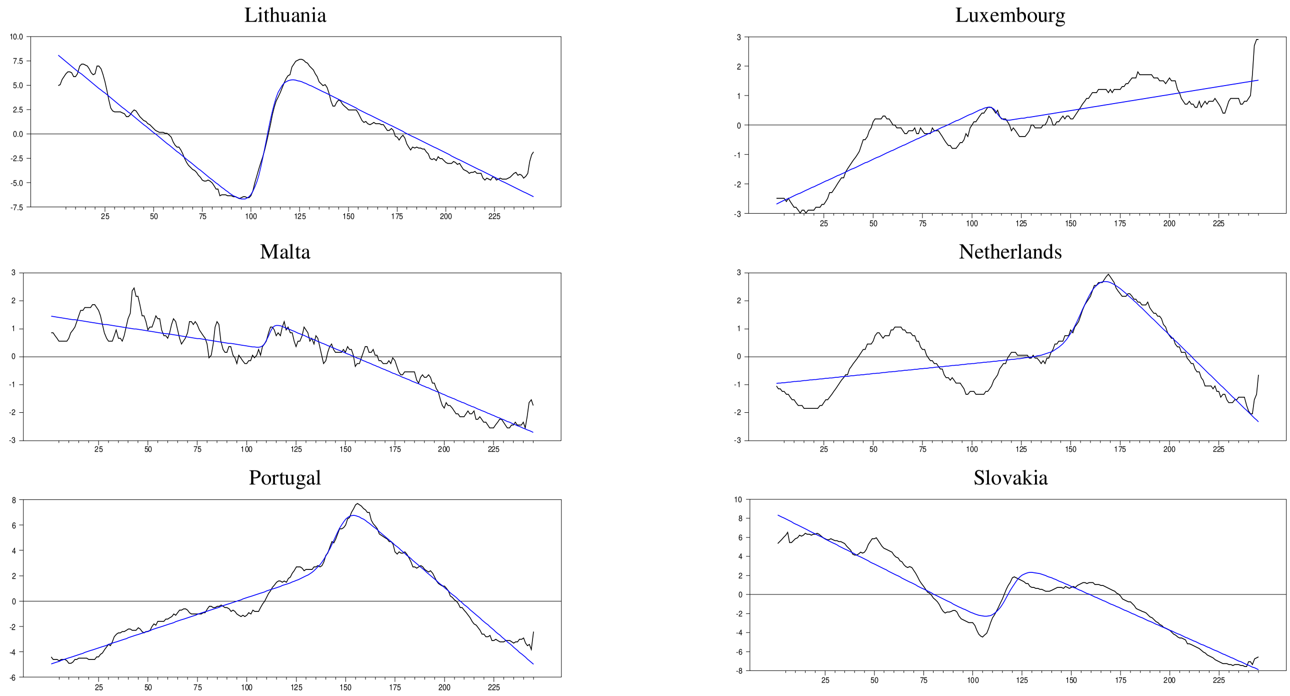
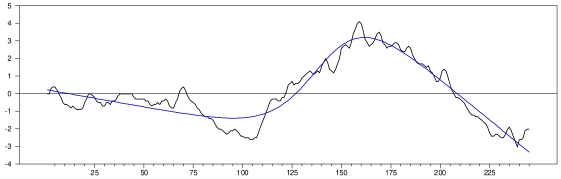


Figure 1. Continued

Slovenia



Spain

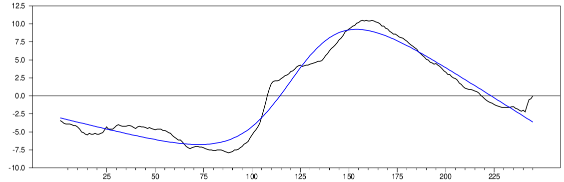
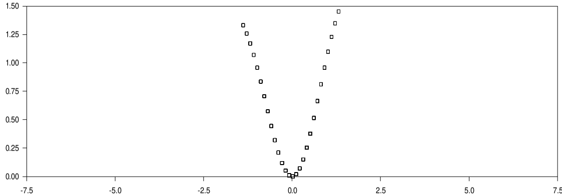
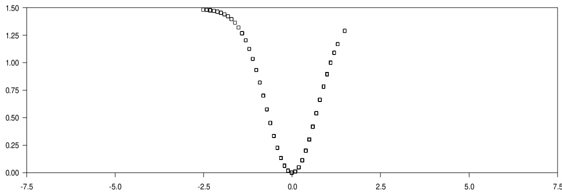


Figure 2. Estimated trend functions (blue line) and unemployment rates (black line)

Austria



Belgium



Cyprus

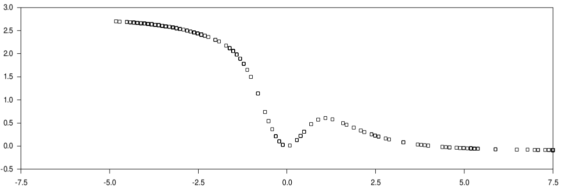


Figure 2. Continued

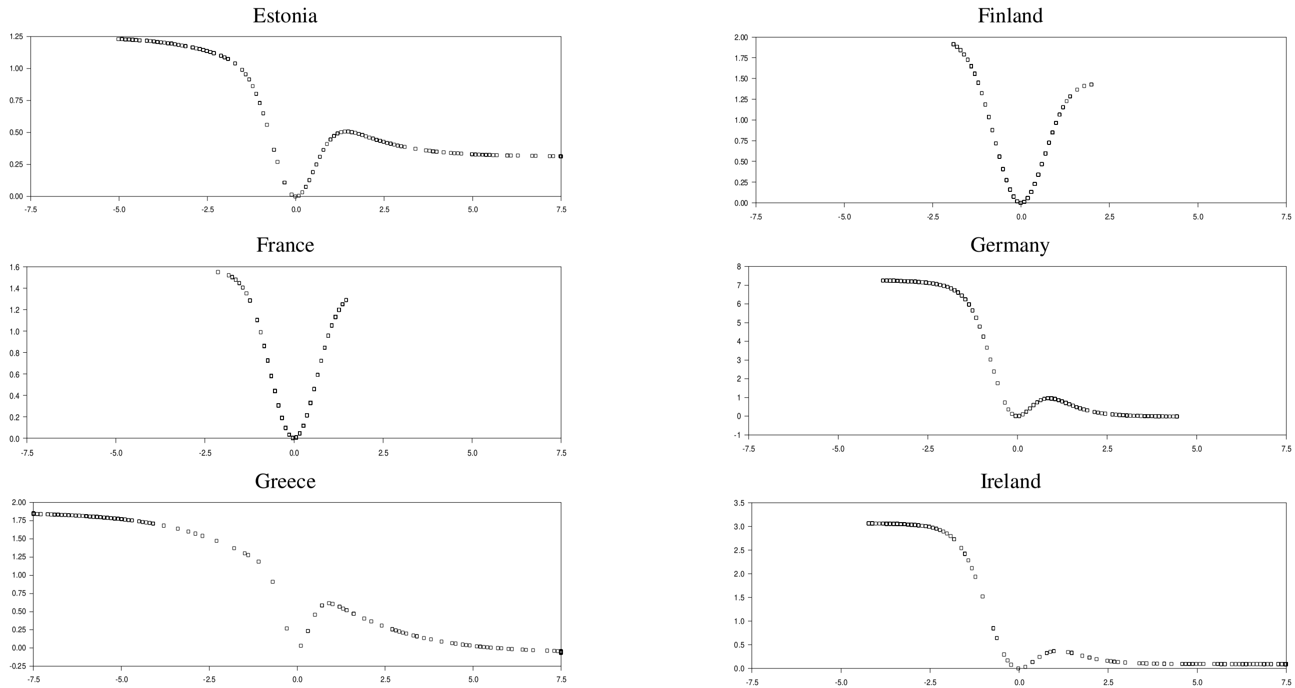


Figure 2. Continued

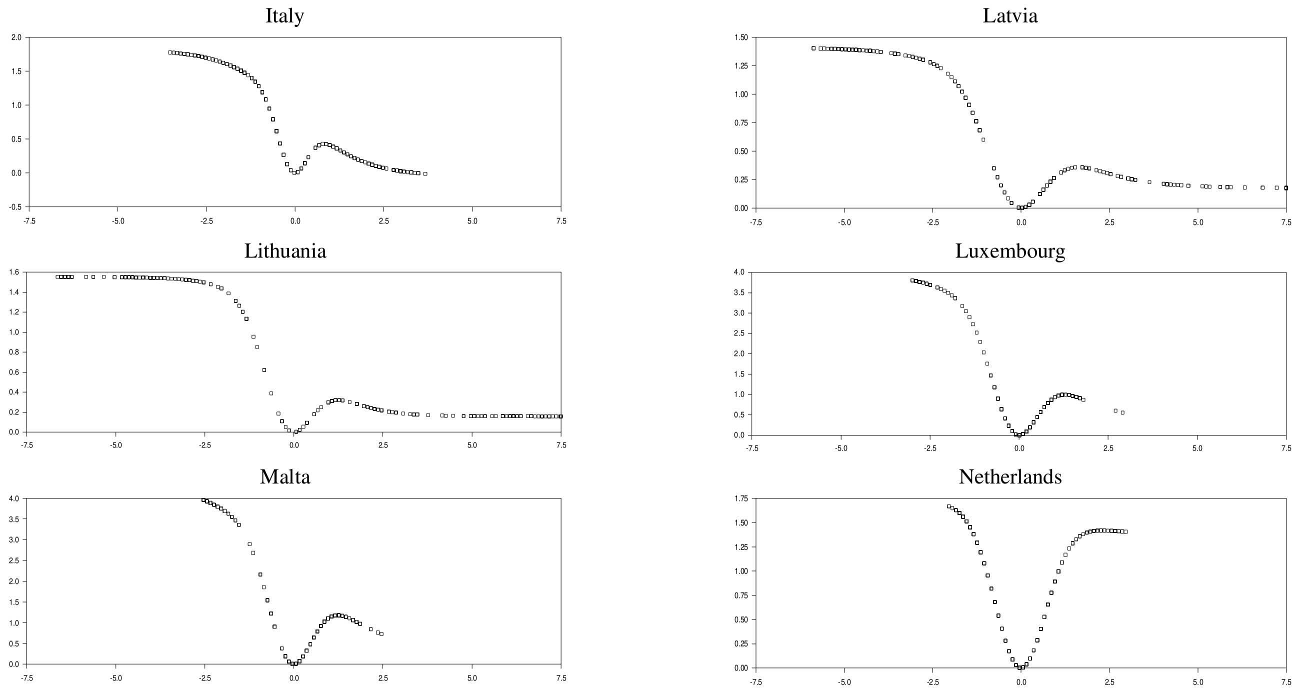
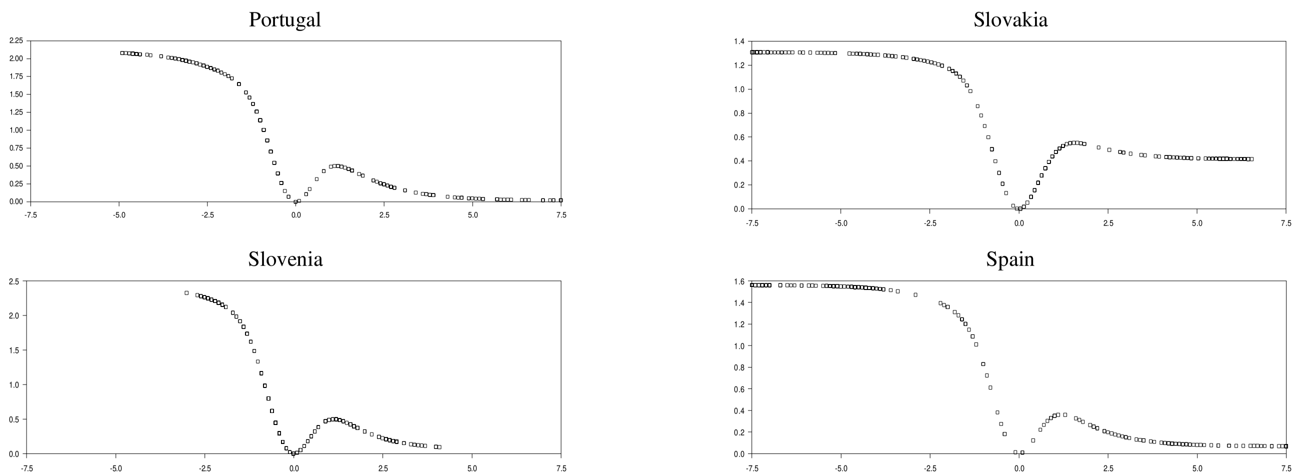


Figure 2. Continued



Technical Annex

Part A: Sieve bootstrap algorithm

All critical values and the power analysis of the proposed test statistics were performed using the RATS software. Tests proposed in this paper can be carried out online using the NONSTAT platform (which uses the R software) by accessing the following link: http://easyfinancetechnology.com/?page_id=578
Password: Nonstat

Sieve Bootstrap Method

Following Ucar and Omay (2009), we apply the sieve bootstrap method to deal with the cross-section dependence problem. The steps of this procedure are as follows:

i. Let $\hat{u}_{i,t}$ denote the demeaned and de-trended series. To de-trend the series, first estimate the nonlinear parameters. The following OLS regression is considered for each entity which allows for different lag orders p_i ;

$$\Delta\hat{u}_{i,t} = \rho_{1i}\hat{u}^3_{i,t-1} + \rho_{2i}\hat{u}^4_{i,t-1} + \sum_{j=1}^{p_i}\psi_{i,j}\Delta\hat{u}_{i,t-j} + \eta_{i,t}$$

ii. The null of no unit root is imposed to generate samples of residuals (Basawa *et al.,* 1991). Errors are estimated as:

$$\eta_{i,t} = \Delta\hat{u}_{i,t} - \sum_{j=1}^{p_i}\varpi_{i,j}\Delta\hat{u}_{i,t-j} \tag{25}$$

iii. Stine (1987) suggests that the residuals have to be centred with

$$\tilde{\eta}_t = \hat{\eta}_t - \frac{1}{(T-p-2)}\sum_{t=p+2}^{T}\hat{\eta}_t \tag{26}$$

where $\hat{\eta}_t = (\hat{\eta}_{1,t}, \hat{\eta}_{2,t}, ..., \hat{\eta}_{N,t})'$ and $p = \max_i(p_i)$. Moreover, we construct $NxT\left[\tilde{\eta}_{i,t}\right]$ matrix from these residuals. We select the residuals column randomly with replacement at a time to preserve the cross-section structure of the errors. The bootstrap residuals are denoted as $\tilde{\eta}^*_{i,t}$ where $t = 1, 2, ..., T^*$ and $T^* = 2T$.

iv. We firs generate stationary bootstrap samples $\Delta\hat{w}^*_{i,t}$ recursively from

$$\Delta u^*_{i,t} = \sum_{j=1}^{p_i} \kappa_{i,j} \Delta u^*_{i,t-j} + \tilde{\eta}^*_{i,t}$$

where the initial values of $\Delta u^*_{i,t-j}$ are set to zero. We then generate the $u^*_{i,t}$ as the partial sum process

$$u^*_{i,t} = \sum_{k=1}^{t} \Delta u^*_{i,k} \tag{28}$$

The bootstrap statistics $\bar{t}^*_{BRNL,j}$ $j = \{A,B,C\}$ are computed for each bootstrap replication by running the regression (keeping in mind that $u^*_{i,t}$ are the demeaned and detrended series)

$$\Delta u^*_{i,t} = \rho_{1,i} u^{3*}_{i,t-1} + \rho_{2,i} u^{4*}_{i,t-1} + \sum_{j=1}^{p_i} \vartheta_{i,j} \Delta u^*_{i,t-j} + v_{i,t} \tag{29}$$

Empirical distribution of these statistics is produced using 2000 replications. Thus, their p-values are generated using this methodology.

Part B: Explanation graphs for Power Analysis

In Figure B.1, graphs were drawn for different speeds of the gamma parameter, which determines the slope of the logistic smooth transition trend. As the gamma transition rate decreases, the LST trend becomes linear for each break size. Therefore, as the pass rate decreases, the power of linear panel unit-root tests increases. Likewise, as the gamma transition rate increases, the power of linear panel unit root tests decreases

In Figure B2, we plot the figures of the transition function $S_{i,t}(\gamma_i, \tau_i)$ for different time dimensions. For the small T=20, the transition function looks like a straight line. As time dimension increases (T=70 or T=100), structural change occurs gradually whilst structural change becomes abrupt for very large T=500. In addition, Figure B.2 shows that although the transition rate parameter gamma is lower as the T size increases, it evolves towards a more abrupt change as the T size increases.

Figure B.1. Changing transition slope with LST trend

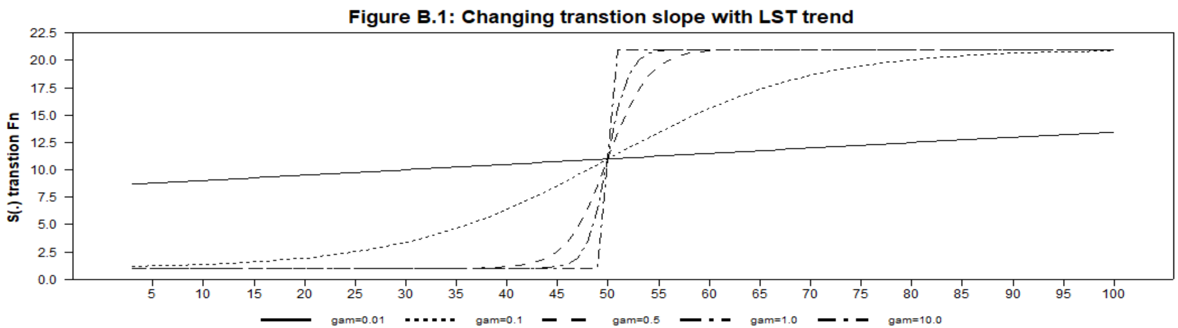
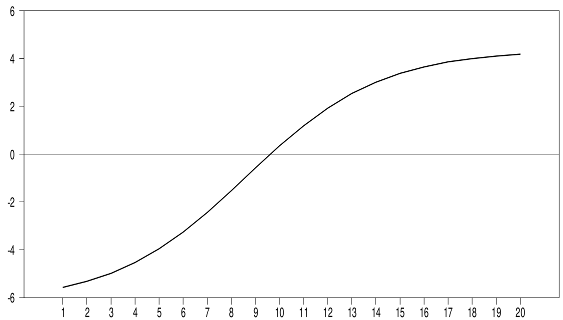
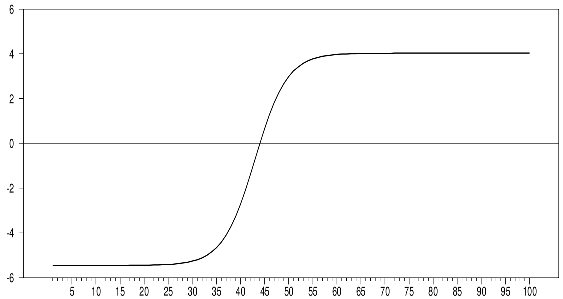


Figure B2. Smooth trend function for different time dimensions

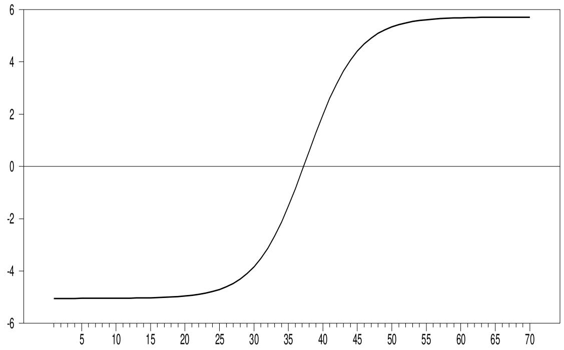


T= 20

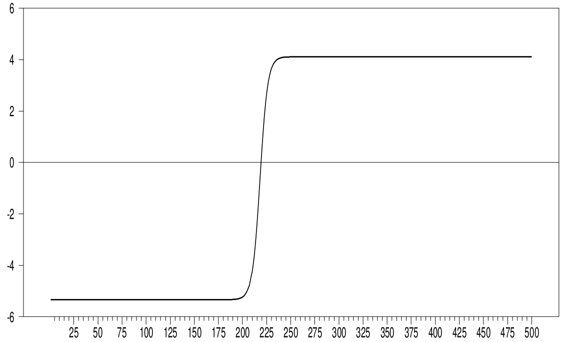


T=100

Figure B2. Continued



T= 70



T=500

In all cases, $\gamma = 0.25$ and $\tau = 0.5$